Записки научных
семинаров ПОМИ
Том 553, 2026 г.

A. A. Matskovskiy, G. L. Zavorokhin

EXISTENCE OF A LOCALIZED WEDGE MODE IN TOPOGRAPHIC WAVEGUIDES WITH A RIGHT-ANGLED TRIANGULAR RIDGE

Abstract. We prove the existence of a localized wedge mode in a homogeneous isotropic elastic topographic waveguide formed by an elastic half-space with an attached triangular ridge whose interior angle is $\frac{\pi}{2}$. Using a variational method, we construct a suitable trial function and show that, in the short-wavelength regime, for all admissible non-zero values of Poisson's ratio, the Rayleigh quotient for this function lies strictly below the threshold of the essential spectrum of the elasticity operator.

Dedicated to the memory of our teacher, Professor V. M. Babich

§1. Introduction

Elastic waves localized near geometric features of a free surface, such as ridges and wedges (see Fig. 1), have long been of interest in seismology, nondestructive testing, and surface acoustic wave technology. Such topographic waveguides can support guided modes whose energy is localized near the ridge edge and whose phase velocity is lower than the Rayleigh-wave velocity c_R of the corresponding flat elastic half-space [1].

The first studies of wedge and topographic waveguides were based mainly on numerical methods [2–5]. Subsequent analytical studies at the physical level of rigor employed asymptotic and approximate methods, including geometrical-acoustics (ray) approaches, thin-plate approximations, and perturbation techniques; see, for example, [6–13]. These works provided qualitative and quantitative descriptions of wedge modes and their dependence on the geometry and material parameters of the elastic solid,

Key words and phrases: topographic waveguide, localized wedge mode, Rayleigh wave, variational method, discrete spectrum, short-wavelength regime.

This work was supported by the Ministry of Science and Higher Education of the Russian Federation (agreement 075-15-2025-344 dated 29/04/2025 for Saint Petersburg Leonhard Euler International Mathematical Institute at PDMI RAS).

in particular, the Poisson ratio $\sigma = \frac{\lambda}{2(\lambda+\mu)}$, where λ and μ are the Lamé constants.

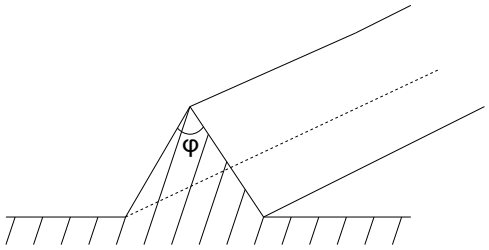


Figure 1. Topographic waveguide.

A rigorous mathematical approach to topographic waveguides was developed by Bonnet–Ben Dhia, Duterte, and Joly [14]. They formulated the problem in terms of a family of self-adjoint spectral problems for the cross-section of the waveguide and related guided modes to discrete eigenvalues below the threshold $c_R^2 k^2$ of the essential spectrum of the elasticity operator, k is the wave number. For triangular topographic perturbations, they proved that, for any prescribed positive integer m, a sufficiently small interior angle φ and a sufficiently large wavenumber k guarantee the existence of at least m guided modes [14].

The result was subsequently extended by Babich [15], who proved the existence of a localized mode for arbitrary acute interior angles $\varphi < \frac{\pi}{2}$ in the high-frequency regime. Thus, the right-angle configuration $\varphi = \pi/2$ is the limiting case not covered by this theorem.

The importance of the angle was already emphasized by Bonnet–Ben Dhia, Duterte, and Joly in their discussion of open questions [14]. They specifically raised the question of the maximum angle below which a localized mode exists, at least at high frequencies. In that work, they noted numerical and semi-analytical indications [16, 17] suggesting that this angle might be slightly larger than $\frac{\pi}{2}$ for some values of the Poisson ratio. They also pointed out the close connection with the problem of localized waves in an infinite elastic wedge, which is mathematically distinct from a topographic waveguide because the latter is a compact perturbation of a half-space [14].

The related spectral problem for an infinite elastic wedge has also been treated rigorously: existence of localized wedge modes has been proved for various ranges of the interior angle φ and material parameters [18–21]. These results, however, concern an unbounded angular domain and therefore differ from the topographic waveguide problem considered here: the infinite wedge mode is dispersion-free, whereas a topographic waveguide with a substrate exhibits dispersion due to the finite size of the ridge, which introduces a characteristic length scale.

Apart from the rigorous spectral-theoretic studies, finite and truncated wedge configurations have also been investigated from the applied point of view. Sokolova et al. [22] studied weak dispersion of wedge acoustic waves caused by deviations from an ideal infinite wedge, including truncation, coatings, and local modifications of the material properties. Vatulyan and Parinova [23] considered elastic waves in a finite-height wedge with a clamped base and analyzed its wave characteristics using approximate variational and Ritz methods. These studies concern finite or perturbed wedge structures but do not address the rigorous existence problem for a triangular ridge attached to an elastic half-space with a free surface.

In the present paper, we consider a homogeneous isotropic elastic half-space with an attached finite triangular ridge whose interior angle is $\varphi = \frac{\pi}{2}$. We prove the existence of a localized waveguide mode in the short-wavelength regime for every admissible non-zero Poisson ratio. The proof is variational. We develop a variational construction adapted to the right-angle geometry, which extends the general variational strategy of [21,24,25] with an ansatz specific to the right-angle case, and establish, for sufficiently large wavenumbers k, a Rayleigh quotient strictly below the essential-spectrum threshold $c_R^2 k^2$. This implies the existence of a discrete eigenvalue of the elasticity operator below the threshold and hence of a localized wedge mode.

§2. Problem Statement. A result due to V. M. Babich

We assume that K is a domain in

$$\mathbb{R}^3 = \{(x_1, x_2, x_3) : x_i \in \mathbb{R},\ i = 1, 2, 3\},$$

described by the inequality $x_2 < f(x_1)$, where $f(x_1)$ is a nonnegative continuous function defined for $x_1 \in \mathbb{R}$ and vanishing outside a finite interval $[l, L]$. Moreover, the graph of $f(x_1)$ for $l \leqslant x_1 \leqslant L$ is assumed to consist of two rectilinear segments $[lV]$ and $[VL]$ ($f(l) = f(L) = 0$, $V = (x_0, f(x_0))$),

$x_0 \in (l, L)$, $f(x_0) > 0$) forming an angle φ at V. The domain K is a topological product, $K = \Omega \times \mathbb{R}$, where Ω is called the cross-section of a homogeneous isotropic elastic topographic waveguide, defined by $x_2 < f(x_1)$ in the (x_1, x_2)-plane.

Let $\mathcal{U} = (\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3)$ be a solution to the system of equations of the theory of elasticity in K with traction-free boundary conditions on ∂K (see [26, Chap. 3]) in the form

$$\mathcal{U}(x_1, x_2, x_3, t) = \mathbb{U}(x_1, x_2, x_3)e^{-i\omega t},$$

where $\omega > 0$ is a given oscillation frequency. Then for the displacement vector function $\mathbb{U} = (\mathbb{U}_1, \mathbb{U}_2, \mathbb{U}_3)$ we obtain the boundary value problem:

$$\begin{aligned} \mathcal{L}(\partial_1, \partial_2, \partial_3)\mathbb{U} &= \rho\omega^2\mathbb{U} \quad \text{in} \quad K, \\ \mathcal{N}(\partial_1, \partial_2, \partial_3)\mathbb{U} &= 0 \qquad \text{on} \quad \partial K, \end{aligned} \tag{1}$$

where $\rho > 0$ is the constant density of the material, and $\partial_m = \frac{\partial}{\partial x_m}$. Without loss of generality, we set $\rho = 1$ throughout the paper. The differential expressions in (1) are defined component-wise by the following relations for $n = 1, 2, 3$:

$$\begin{aligned} \big(\mathcal{L}(\partial_1, \partial_2, \partial_3)\mathbb{U}\big)_n &= -\partial_m \sigma_{nm}[\mathbb{U}], \\ \big(\mathcal{N}(\partial_1, \partial_2, \partial_3)\mathbb{U}\big)_n &= \sigma_{nm}[\mathbb{U}]\nu_m, \end{aligned} \tag{2}$$

where $\nu = (\nu_1, \nu_2, \nu_3)$ is the unit outward normal vector to ∂K (obviously, $\nu_3 = 0$). Here and henceforth, summation from 1 to 3 is implied over repeated indices.

In (2), $\sigma_{nm}[\mathbb{U}]$ denotes the components of the stress tensor

$$\sigma_{nm}[\mathbb{U}] = \lambda\, \partial_\ell \mathbb{U}_\ell \delta_{n,m} + \mu\big(\partial_m \mathbb{U}_n + \partial_n \mathbb{U}_m\big),$$

where $\delta_{n,m}$ is the Kronecker delta, and the Lamé constants λ and μ satisfy the inequalities $\lambda + \frac{2}{3}\mu > 0$ and $\mu > 0$.

The solution to problem (1) is sought in the form

$$\mathbb{U}(x_1, x_2, x_3) = u(x_1, x_2)e^{ikx_3},$$

which describes an elastic wave propagating along the ridge edge of the waveguide K, where $k > 0$ is the wave number. The problem then reduces to the system

$$\begin{aligned} \mathcal{L}(\partial_1, \partial_2, ik)u &= \omega^2 u \quad \text{in} \quad \Omega, \\ \mathcal{N}(\partial_1, \partial_2, ik)u &= 0 \quad \text{on} \quad \partial\Omega. \end{aligned} \tag{3}$$

Henceforth, it is assumed that in all formulae, ∂_3 should be replaced by ik.

Since the wedge modes are localized in the vicinity of the edge, the vector function $u = (u_1, u_2, u_3)$ should decay at infinity.

The variational (weak) formulation of the problem (3) refers to the integral identity

$$a_\Omega(ik; u, v) := \int\limits_\Omega \sigma_{nm}[u]\overline{\partial_m v_n}\, dx_1 dx_2 = \omega^2 (u, v)_\Omega, \quad v \in \left(H^1(\Omega)\right)^3, \quad (4)$$

where bar stands for complex conjugation and $(\cdot, \cdot)_\Omega$ for the natural inner product in $\left(L^2(\Omega)\right)^3$. The quadratic form $a_\Omega(ik; u, u)$ in $\left(L^2(\Omega)\right)^3$ with domain $\left(H^1(\Omega)\right)^3$ is symmetric, positive and closed. Therefore, the boundary value problem (3) is associated with an unbounded positive definite self-adjoint operator $\mathcal{A}(ik)$ in the space $\left(L^2(\Omega)\right)^3$ (see, e.g., [27, Chap. 10]).

If $u \in (L^2(\Omega))^3$ is a nontrivial eigenfunction of the operator $\mathcal{A}(ik)$, $k > 0$, then the function $\mathcal{U}(x_1, x_2, x_3, t) = u(x_1, x_2)e^{-i\omega t + ikx_3}$, $\omega > 0$, will be called *a localized wedge mode*. This paper provides a rigorous proof of the existence of localized wedge modes.

According to the variational principle, the lower bound of the spectrum of the operator $\mathcal{A}(ik)$ is equal to $\inf\limits_{(H^1(\Omega))^3} \Phi(u)$, where

$$\Phi(u) := \frac{a_\Omega(ik; u, u)}{(u, u)_\Omega} \quad (5)$$

is the Rayleigh quotient.

It is shown in [14] that the essential spectrum of this operator coincides with the ray $[c_R^2 k^2, +\infty)$. Consequently, only discrete spectrum may occur below the threshold $\omega_R^2 = c_R^2 k^2$. If this spectrum is non-empty, the corresponding eigenvector generates a localized wedge mode propagating along the edge with a velocity less than c_R.

To prove the existence of the discrete spectrum, it suffices to present a function $u^{\text{test}} \in (H^1(\Omega))^3$ such that

$$\Phi(u^{\text{test}}) < c_R^2 k^2. \quad (6)$$

In [15], to construct this test function, the coordinate system was introduced as shown in Fig. 2. On the interval $[0, d]$, the part of the boundary $\partial\Omega$ that does not lie on the X_1-axis is the rectilinear segment described by the equation $X_2 = X_1 \tan(\varphi), 0 < \varphi < \frac{\pi}{2}$.

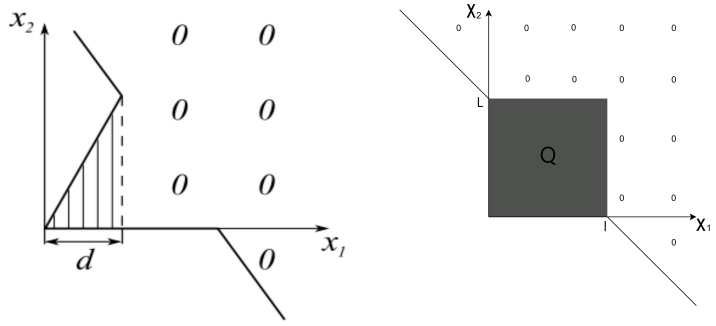


Figure 2. Waveguide cross-section. Domains Ω_1 and Q.

The test function was chosen as

$$u_k^{\text{test}}(X_1, X_2) = \begin{cases} \eta(X_1) u^R(X_2), & \text{in} \quad \Omega_1, \\ 0, & \text{in} \quad \Omega \setminus \Omega_1, \end{cases} \tag{7}$$

where Ω_1 is the triangle (see Fig. 2):

$$0 \leqslant X_1 \leqslant d, \quad 0 \leqslant X_2 \leqslant X_1 \tan\varphi,$$

$$\eta(X_1) = \begin{cases} 1, & \text{if} \quad 0 \leqslant X_1 < d', \\ 0, & \text{if} \quad X_1 > d, \end{cases} \tag{8}$$

is the cut-off function defined for $X_1 \geqslant 0$,

$$0 < d' < d, \quad d, d' = \text{const}, \quad \eta \geqslant 0, \quad \frac{d\eta}{dX_1} \leqslant 0, \quad \eta \in C^\infty(\mathbb{R}_+),$$

and

$$\boldsymbol{U}^R = e^{-i\omega t + ikx_3} u^R(X_2), \tag{9}$$

is the Rayleigh wave [1] traveling in the direction (0,0,1) and leaving the plane $X_2 = 0$ free of stresses.

It turns out that

$$\Phi(u_k^{\text{test}}) = c_R^2 k^2 - ck + O(1), \quad k \to +\infty, \tag{10}$$

where $c > 0$ for $\varphi < \frac{\pi}{2}$. Then (10) implies (6) for k sufficiently large. This proves the existence of a localized wedge mode for $\varphi < \frac{\pi}{2}$ and for all admissible values of the Poisson's ratio σ.

In the right-angle case ($\varphi = \pi/2$), the constant c in (10) vanishes, rendering the given proof inapplicable (see [18, Remark 2], [19, 21]). Our main result, which covers this case, is formulated in the next section.

§3. A generalization of V. M. Babich's result

Theorem 1. *Let $\varphi = \pi/2$ and $\sigma \neq 0$. Then $\mathcal{A}(ik)$ has at least one eigenvalue in the interval $(0, c_R^2 k^2)$ for all sufficiently large k.*

Proof. As in [15], we place the origin at the ridge vertex V and direct the X_1- and X_2-axes along the two sides of the ridge cross-section. The transformation from (x_1, x_2) to (X_1, X_2) is a translation to V followed by a rotation.

Let $Q = \{(X_1, X_2) \in \mathbb{R}^2 : 0 \leqslant X_1 \leqslant l,\ 0 \leqslant X_2 \leqslant L\}$ be the fixed rectangle (see Fig. 2). We introduce two smooth cut-off functions. Let $\eta \in C^\infty(\mathbb{R}_+)$ be a non-increasing function satisfying

$$\eta(X_1) = 1, \quad 0 \leqslant X_1 < l', \qquad \eta(X_1) = 0, \quad X_1 > l,$$

where $0 < l' < l$, $\quad l', l = \text{const}$, and let $\zeta \in C^\infty(\mathbb{R}_+)$ be a non-increasing function satisfying

$$\zeta(X_2) = 1, \quad 0 \leqslant X_2 < L', \qquad \zeta(X_2) = 0, \quad X_2 > L,$$

where $0 < L' < L$, $\quad L', L = \text{const}$. Both cut-off functions are assumed to be non-negative, $\eta \geqslant 0$, $\zeta \geqslant 0$.

Define the test function by

$$v_k^{\text{test}}(X_1, X_2) = \begin{cases} \eta(X_1)\zeta(X_2)u^R(X_2) + k^{-1/2}W(kX_1, kX_2), & (X_1, X_2) \in Q, \\ 0, & (X_1, X_2) \in \Omega \setminus Q, \end{cases} \tag{11}$$

where k is a large parameter, and $u^R(X_2) = (0, u_2^R(X_2), u_3^R(X_2))$ is the Rayleigh wave profile (9) corresponding to the wavenumber k. We write $u^R(X_2) = \mathcal{U}^R(kX_2)$, where $\mathcal{U}^R$ is the fixed exponentially decaying profile.

The function $W = (W_1, W_2, W_3)$, to be chosen below, is smooth up to the boundary of the closed first quadrant $\{Y_1 \geqslant 0,\ Y_2 \geqslant 0\}$ and $W(Y_1, Y_2)$ has compact support contained in $[0, l'] \times [0, L']$. The trace of W on $Y_1 = 0$ is not assumed to vanish. The support of $W(kX_1, kX_2)$ is contained in $[0, l'/k] \times [0, L'/k]$. Hence, for all sufficiently large k, both cut-off functions η, ζ are identically equal to one on this support. In particular, on the support of $W(kX_1, kX_2)$, the first term in (11) coincides with $u^R(X_2)$. The cut-off functions ensure that the zero extension in (11) introduces no

jump across the artificial sides $X_1 = l$ and $X_2 = L$ of Q, which lie in the interior of Ω. Thus, $v_k^{\text{test}} \in (H^1(\Omega))^3$.

Substituting (11) into the Rayleigh quotient (5), it suffices to show that

$$\Phi(v_k^{\text{test}}) < c_R^2 k^2,$$

which is equivalent to

$$\mathcal{B}_k(v_k^{\text{test}}, v_k^{\text{test}}) < 0, \tag{12}$$

where

$$\mathcal{B}_k(u, v) := a_\Omega(ik; u, v) - c_R^2 k^2 (u, v)_\Omega.$$

Put

$$v_{0,k}(X_1, X_2) = \eta(X_1)\zeta(X_2)u^R(X_2), \qquad W_k(X_1, X_2) = W(kX_1, kX_2).$$

Here and below, the subscript k in W_k denotes the scaled function. Numerical subscripts denote vector components, so that $W_{k,j}(X_1, X_2) = W_j(kX_1, kX_2)$. Then

$$v_k^{\text{test}} = v_{0,k} + k^{-1/2} W_k.$$

Expanding the quadratic form in (12) yields

$$\mathcal{B}_k(v_k^{\text{test}}, v_k^{\text{test}}) = \mathcal{B}_k(v_{0,k}, v_{0,k}) + 2k^{-1/2} \operatorname{Re} \mathcal{B}_k(v_{0,k}, W_k) + k^{-1} \mathcal{B}_k(W_k, W_k). \tag{13}$$

We first estimate the first term in (13). Introduce the constants

$$I_\eta = \int_0^\infty \eta^2(X_1)\, dX_1, \quad J_\eta = \int_0^\infty |\eta'(X_1)|^2\, dX_1, \quad I_R = \int_0^\infty |\mathcal{U}^R(Y_2)|^2\, dY_2.$$

Recall that $u^R(X_2)$ denotes the Rayleigh wave profile, i.e., the solution of the wave propagation problem in the elastic half-space $X_2 > 0$ along the traction-free boundary $X_2 = 0$, decaying as $X_2 \to +\infty$, see [26, Ch. 3]. Applying the product rule to $\eta(X_1)\zeta(X_2)u^R(X_2)$ and using the integral identity (4) for the Rayleigh wave $u^R(X_2)$, we obtain

$$\begin{aligned}\mathcal{B}_k(v_{0,k}, v_{0,k}) = \mu J_\eta \int_0^\infty \zeta^2(X_2) \big(|u_2^R(X_2)|^2 + |u_3^R(X_2)|^2\big)\, dX_2 \\ + I_\eta \int_0^\infty |\zeta'(X_2)|^2 \left[(\lambda + 2\mu)|u_2^R(X_2)|^2 + \mu |u_3^R(X_2)|^2\right] dX_2.\end{aligned}$$

Here the terms containing η' and ζ' enter separately, and no mixed $\eta'\zeta'$-term arises, since the first component of u^R vanishes.

Since $\mathcal{U}^R$ decays exponentially, there exists $\beta > 0$ such that $|\mathcal{U}^R(Y_2)| \leqslant Ce^{-\beta Y_2}$. The condition $\zeta(X_2) = 1$ for $0 \leqslant X_2 \leqslant L'$ therefore gives

$$\int_0^\infty \zeta^2(X_2)|u^R(X_2)|^2\,dX_2 = \frac{1}{k}\int_0^\infty \zeta^2(Y_2/k)|\mathcal{U}^R(Y_2)|^2\,dY_2 = \frac{I_R}{k} + O\left(\frac{e^{-2\beta kL'}}{k}\right).$$

Moreover, since ζ' is supported in $[L', L]$, we have

$$\int_0^\infty |\zeta'(X_2)|^2\left[(\lambda+2\mu)|u_2^R(X_2)|^2 + \mu|u_3^R(X_2)|^2\right]\,dX_2 = O\left(\frac{e^{-2\beta kL'}}{k}\right).$$

Consequently,

$$\mathcal{B}_k(v_{0,k}, v_{0,k}) = \frac{\mu J_\eta I_R}{k} + O\left(\frac{e^{-2\beta kL'}}{k}\right) = O(k^{-1}). \tag{14}$$

For the last term in (13), the change of variables $Y_1 = kX_1$, $Y_2 = kX_2$ shows that each derivative $\partial_1, \partial_2, \partial_3$ contributes a factor k, whereas the Jacobian yields k^{-2}. Since W has fixed compact support, it follows that

$$a_\Omega(ik; W_k, W_k) = O(1), \qquad k^2(W_k, W_k)_\Omega = O(1),$$

and hence

$$k^{-1}\mathcal{B}_k(W_k, W_k) = O(k^{-1}). \tag{15}$$

We now evaluate the mixed term in (13). Since $\eta = \zeta = 1$ on the support of W_k for all sufficiently large k, we have

$$\mathcal{B}_k(v_{0,k}, W_k) = \mathcal{B}_k(u^R, W_k).$$

Using the integral identity (4) and the fact that the Rayleigh wave $u^R(X_2)$ satisfies

$$\mathcal{L}(0, \partial_2, ik)u^R = c_R^2 k^2 u^R \quad \text{for } X_2 > 0, \qquad \mathcal{N}(0, \partial_2, ik)u^R = 0 \quad \text{at } X_2 = 0,$$

we obtain

$$\begin{aligned}\mathcal{B}_k(u^R, W_k) &= a_\Omega(ik; u^R, W_k) - c_R^2 k^2 (u^R, W_k)_\Omega \\ &= \int_{\partial\Omega} \sigma_{nm}[u^R]\nu_m \overline{W_{k,n}}\,d\Gamma = -\int_0^\infty \sigma_{11}[u^R](0, X_2)\overline{W_{k,1}(0, X_2)}\,dX_2.\end{aligned} \tag{16}$$

Since the stress scales linearly with k, there exists a function Σ_{11}, independent of k, such that

$$\sigma_{11}[u^R](0, X_2) = k\,\Sigma_{11}(kX_2).$$

Changing variables to $Y_2 = kX_2$ in (16), we obtain

$$\mathcal{B}_k(u^R, W_k) = -\int_0^\infty \Sigma_{11}(Y_2)\,\overline{W_1(0, Y_2)}\,dY_2. \tag{17}$$

Define

$$B[W] = -\operatorname{Re}\int_0^\infty \Sigma_{11}(Y_2)\,\overline{W_1(0, Y_2)}\,dY_2.$$

Combining (17) with the estimates for the first and last terms in (13), given by (14) and (15), respectively, we arrive at

$$\mathcal{B}_k(v_k^{\text{test}}, v_k^{\text{test}}) = 2k^{-1/2}B[W] + O(k^{-1}). \tag{18}$$

By construction, $B[W]$ is independent of k. The function $\Sigma_{11}(Y_2)$ is non-zero for all $Y_2 \geqslant 0$, provided that the Poisson's ratio $\sigma \neq 0$. Consequently, by choosing the function W, $B[W]$ in (18) can be made negative, and thus inequality (12) will be satisfied for sufficiently large k. This completes the proof.

Note that if the Poisson's ratio $\sigma = 0$, then the Rayleigh wave (9) would satisfy the traction-free boundary conditions on the face $X_1 = 0$ as well as on $X_2 = 0$. In this case, we were unable to prove the existence of a localized waveguide mode, and we conjecture that it does not exist, see also [18, 19, 21].

Moreover, the variational construction presented here suggests extensions to a wider class of cross-sectional domains in which a flat boundary segment supporting a Rayleigh wave meets another boundary segment at a right angle. The same argument applies to such domains, provided that the boundary contribution in this construction remains non-zero and the essential-spectrum threshold $c_R^2k^2$ is preserved. This is consistent with Babich's observation in [15, §5] that the result for acute angles extends to a wide class of domains whose boundary contains an angle and whose essential spectrum has the same threshold.

References

1. Lord Rayleigh (J. W. Strutt), *On Waves Propagating Along the Plane Surface of an Elastic Solid*. — Proc. London Math. Soc. **17** (1885), 4–11.
2. P. E. Lagasse, *Analysis of a dispersion-free guide for elastic waves*. — Electron. Lett. **8**, No. 15 (1972), 372–373.
3. P. E. Lagasse, *Higher-order finite element analysis of topographic guides supporting elastic surface waves*. — J. Acoust. Soc. Amer. **53** (1973), 1116–1122.
4. P. E. Lagasse, I. M. Mason, E. A. Ash, *Acoustic surface waveguides analysis and assessment*. — IEEE Trans. Microwave Theory Tech. **21** (1973), 225–236.
5. S. L. Moss, A. A. Maradudin, S. L. Cunningham, *Vibrational Edge Modes for Wedges with Arbitrary Interior Angles*. — Phys. Rev. B, **8**, No. 6 (1973), 2999–3008.
6. A. A. Maradudin, *Surface acoustic waves on rough surfaces*. — In: Recent Developments in Surface Acoustic Waves, eds. D. F. Parker and G. A. Maugin (1988), 100–128.
7. V. V. Krylov, *Geometrical-acoustics approach to the description of localized vibrational modes of an elastic wedge*. — Sov. Phys. Tech. Phys. **35** (1990), 137–140.
8. H. F. Tiersten, D. Rubin, *On the Fundamental Antisymmetric Mode of the Wedge Guide*. — Proc. IEEE Ultrason. Symp. (1974), 117–120.
9. J. McKenna, G. D. Boyd, R. N. Thurston, *Plate theory solution for guided flexural acoustic waves along the tip of a wedge*. — IEEE Trans. on Sonics and Ultrasonics, **21**, No. 3 (1974), 178–186.
10. V. G. Mozhaev, *Ray theory of wedge acoustic waves*. — Moscow University Physics Bulletin **44**, No. 5 (1989), 38–42.
11. D. F. Parker, *Elastic wedge waves*. — J. Mech. Phys. Solids **40**, No. 7 (1992), 1583–1593.
12. V. G. Mozhaev, *Unexpectable relations in the theory of surface wave propagation in topographical slightly perturbated media*. — In: Conference "Acoustoelectronic Surface Wave Devices for Signal Processing", Cherkassy, USSR, Proceedings. Moscow, VINITI (1991), 12–13, [in Russian].
13. S. V. Biryukov, Yu. V. Gulyaev, V. V. Krylov, V. P. Plessky. *Surface Acoustic Waves in Inhomogeneous Media*. Springer-Verlag, Berlin–Heidelberg–New York (1995).
14. A. S. Bonnet-Ben Dhia, J. Duterte, P. Joly, *Mathematical analysis of elastic surface waves in topographic waveguides*. — MMMAS **9**, No. 5 (1999), 755–798.
15. V. M. Babich, *A class of topographical waveguides*. — Algebra Analysis, **22**, No. 1 (2010), 98–107.
16. J. Duterte, *Ondes de surface guidées par une perturbation locale d'un demi-espace élastique homogène*. — Ph.D. thesis, Université Paris, Paris (1995).
17. J. Duterte, P. Joly, *A Numerical Method for Surface Waves in a Cylindrically Perturbed Elastic Half-Space. Part* 1: *Construction and Analysis*. — SIAM J. Appl. Math. **59**, No. 5 (1999), 1559–1588.
18. I. V. Kamotskiĭ, *Surface wave running along the edge of an elastic wedge*. — Algebra Analysis **20**, No. 1 (2008), 86–92.
19. G. L. Zavorokhin, A. I. Nazarov, *On elastic waves in a wedge*. — Zap. Nauchn. Semin. POMI **380** (2010), 45–52.
20. A. Nazarov, S. Nazarov, G. Zavorokhin, *On symmetric wedge mode of an elastic solid*. — Europ. J. Appl. Math. **33**, No. 2 (2022), 201–223.

21. A. A. Matskovskiy, G. L. Zavorokhin, *On the existence of a localized waveguide mode in a rectangular elastic wedge*. — IEEE Proceedings of Days on Diffraction, St. Petersburg (2025), 136–139.
22. E. S. Sokolova, A. S. Kovalev, R. Timler, A. P. Mayer, *On the dispersion of wedge acoustic waves*. — Wave Motion **50**, No. 2 (2013), 233–245.
23. A. O. Vatulyan, L. I. Parinova, *On the elastic waves propagating along the edge of the wedge with small opening angle*. — In: Advanced Materials – Techniques, Physics, Mechanics and Applications, Springer Proceedings in Physics, **193** (2017), 309–317.
24. I. V. Kamotskiĭ, S. A. Nazarov, *Elastic waves localized near periodic families of defects*. — Dokl. Physics **368**, No. 6 (1999), 771–773.
25. S. A. Nazarov, *Variational and Asymptotic Methods for Finding Eigenvalues below the Continuous Spectrum Threshold*. — Siberian Math. J. **51**, No. 5 (2010), 866–878.
26. L. D. Landau, E. M. Lifshitz, *Course of Theoretical Physics*, Vol. 7, Theory of Elasticity. 3rd Edition, Elsevier (2012).
27. M. S. Birman, M. Z. Solomjak, *Spectral theory of self-adjoint operators in Hilbert space*. 2nd edition, revised and extended. Lan', St. Petersburg 2010.

St. Petersburg Department
of V. A. Steklov Institute of Mathematics
of the Russian Academy of Sciences,
27, Fontanka, 191023 St. Petersburg, Russia
E-mail: zavorokhin@pdmi.ras.ru, androbasrm@rambler.ru

Поступило 6 октября 2026 г.