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FIELD THEORETIC RENORMALIZATION GROUP IN A MODEL OF RANDOM WALK ON A RANDOM SURFACE

Аннотация. Using the problem of random walk on a random rough surface, we show how neglecting a formally infrared irrelevant (in the sense of Wilson) term in the corresponding action functional makes the regime of non-trivial infrared asymptotic behaviour undetectable. By taking that term into account and employing a nonconventional renormalization scheme in the real d-dimensional space, we establish the "weak" scaling behaviour of the Green's functions with two coexisting different expressions for the critical dimension of time/frequency.

Dedicated to N. M. Bogoliubov
on the occasion of his 75th birthday

§1. Introduction

Random walks are ubiquitous in Nature and provide a classical area of research in mathematics and statistical physics. Besides the ordinary random walk (ordinary diffusion in macroscopic description) they involve problems as disparate as self-avoiding random walk, branching and annihilating walks, vicious and friendly walkers, continuous time random walks and Lévy flights, random walks in random media and so on; see, e.g. [1–21] and references therein.

Nikolai Mikhailovich Bogolyubov with his collaborators made important contribution to understanding mathematical and theoretical aspects of statistical physics and quantum field theory, in particular, in development and applications of the Quantum inverse scattering method [22, 23]. In particular, N.M. and his coauthors invested much into the problem of self-avoiding paths and random walks of vicious and friendly walkers,

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related to integrable models of certain quantum spin chains; see [24–28] and references therein.

The ordinary random walk is characterized by the well-known power law $R(t) \sim t^{1/2}$ for the mean-square displacement of a particle's location (valid for any spatial dimension d). For the case of many independent particles it can be interpreted as the spreading law of a cloud of walking particles.

Diffusion processes and particles' transfer in complex environments such as random, disordered, inhomogeneous, porous or turbulent media demonstrate numerous interesting features: emergence of anomalous sub- and superdiffusion with strongly non-Brownian spreading laws [8, 11, 13, 17], relation to the origin of the flicker noise with the spectrum $1/f^{\alpha}$ [2, 3], trapping [10, 15] and so on.

The most indicative characteristics of a random walk (or diffusion) is the aforementioned spreading law. In the model of a one-dimensional discrete random walk in a random medium due to Sinai [1], it takes on the form $R(t) \sim \log^2 t$, while in other models the typical behaviour is $R(t) \sim t^{\alpha}$ with $\alpha < 1/2$ (subdiffusion) or $\alpha > 1/2$ (superdiffusion); see [8] for general discussion and references.

In a continuous formulation, simple random walks are described by certain Langevin-type stochastic equations, which give rise to macroscopic diffusion-type partial differential equations for the corresponding probability distribution function. Such equations also become stochastic when a random walk proceeds in a random medium. In some cases, they can be formulated as certain field-theoretic models and can be studied using the field-theoretic tools, especially the renormalization group (RG), see, e.g. monograph [29], a recent review [30] and the literature cited therein.

The subject of the present paper is a long-time, large-scale behaviour of a random walk on a random surface. The latter can be considered as a kind of random medium and the study of its stochastic growth has a long history itself [31–39]. This interest is explained by the fact that kinetic roughening of randomly growing surfaces has numerous examples that span the range from deposition of a substance or bacterial colony growth to an urban landscape evolution.

Our pervious research [40–42] covered surfaces described by the (generalized) Edwards–Wilkinson ensemble [32] and the celebrated Kardar–Parisi–Zhang (KPZ) equation [33], while the random walks were considered in a uniform (constant) gravitational field. The RG approach revealed interesting long-range asymptotic behaviour in the infrared (IR) range (large scales and long times), differing from that of an ordinary random walk.

Conservation laws play significant role in dynamical critical phenomena, so that there is a special family of equilibrium dynamical models with a conserved order parameter and/or energy density; see, e.g. Chapter 5 in the monograph [29]. Thus, various generalizations of the KPZ model with local conservation of the height field were introduced and studied in [43–48]. It was shown that they define new universality classes of critical behaviour different from that of the canonical KPZ model.

In this paper we present some preliminary basic results of our study of the random walk on a random rough surface described by the simplest conserved KPZ model introduced in [43], the so-called CKPZ model. The analysis has shown that the problem exhibits rather nontrivial features, most interesting probably from the theoretical point of view: appearance of a certain term (composite operator) which is formally IR irrelevant (in the sense of Wilson) but contributes essentially to the IR behaviour of the Green's functions; hence, insufficiency of the standard RG treatment with dimensional regularization and minimal subtraction scheme and the necessity to apply a non-conventional RG scheme in the real d-dimensional space in which that irrelevant operator is treated from the beginning on equal footing with the relevant terms.

The resulting scaling behaviour of the Green's functions demonstrates non-conventional scaling behaviour with two asymptotic dispersion laws (or, in other words, two different expressions for the critical dimension of time/frequency).

Admittedly, all these issues have been separately encountered earlier in various contexts and problems: dangerous irrelevant operator/variable [49, 50], RG in the real d-dimensional space with non-minimal subtraction [51] in the advanced formulation we apply here [52, 53], fair treatment of an irrelevant operator on equal footing with the others [54, 55] and the weak scaling with a non-unique dimension of frequency [56]. However, it is surprising to find them all deeply connected with each other in a problem that seems quite straightforward at a first glance.

Detailed report will be published elsewhere [57], here we only give a brief account of the listed issues emphasizing the logical steps of our research that ultimately led to the final scaling expressions for the Green's functions and spreading law.

§2. The model and its analysis

Consider a random walker on a d-dimensional surface embedded into the $(d+1)$-dimensional space. The surface height $h(x)$ and the coordinates $x = \{t, \mathbf{x}\}$, $\mathbf{x} = \{x_i\}$, $i = 1, \dots, d$, determine the walker's location in time and space. If F is an external drift field then the stochastic equation of motion for the walker is

$$\partial_t x_i = F_i(x) + \zeta_i, \quad \langle \zeta_i(t)\zeta_j(t')\rangle = 2\kappa_0 \delta(t - t'),$$

where $\zeta_i = \zeta_i(t)$ is a Gaussian noise with zero mean, $\kappa_0 > 0$ is the diffusion coefficient and $\partial_t = \partial/\partial t$ is derivative over the time variable t. Let us pass to the Fokker–Planck equation for the density field of the walkers $P(x)$:

$$\partial_t P + \partial_i \left(F_i - \kappa_0 \partial_i\right) P = 0, \tag{1}$$

where $\partial_i = \partial/\partial x_i$ (note that summation over repeated tensor indices is implied). We assume that a constant gravitational field results in the drift $F_i = -\gamma_0 \partial_i h$ with positive γ_0 (see, e.g. [41]).

At the same time the surface undergoes kinetic roughening described by the CKPZ equation [43]:

$$\partial_t h = -\partial^2 \left\{\nu_0 \partial^2 h + (\lambda_0/2)(\partial_i h)(\partial_i h)\right\} + \eta, \tag{2}$$

where $\partial^2 = \partial_i \partial_i$, ν_0 is a positive coefficient, λ_0 contains the charge and $\eta = \eta(x)$ is another Gaussian random noise with zero mean and the pair correlation function

$$\langle \eta(x)\eta(x')\rangle = -D_0 \partial^2 \delta(x - x'), \quad D_0 > 0. \tag{3}$$

Altogether, all the equations combined read

$$\partial_t P = \kappa_0 \partial^2 P + \gamma_0 \partial_i (P \partial_i h). \tag{4}$$

Here is where the crux of the matter lies: it is quite natural to turn equation (4) in the following form:

$$\xi_0 \partial_t P = \partial^2 P + \alpha_0 \partial_i (P \partial_i h) \tag{5}$$

with $\xi_0 = 1/\kappa_0$ and $\alpha_0 = \gamma_0/\kappa_0$. The reason for that is we want to capitalize on IR irrelevance of the parameter ξ_0, and put $\xi_0 = 0$ in practical calculations later on. Indeed, let us set $D_0 = 1$ and rewrite the problem (2), (3), (5) as a field theory following a well-established De Dominicis–Janssen formalism (see, e.g. Section 5.3 in [29]):

$$S(\Phi) = -\frac{h'\partial^2 h'}{2} + h'\left[-\partial_t h - \partial^2\left(\nu_0\partial^2 h + \frac{1}{2}\lambda_0(\partial h)^2\right)\right] \\ + \theta'\left[-\xi_0\partial_t\theta + \partial^2\theta\right] - \alpha_0(\partial\theta')(\partial h)\theta. \qquad (6)$$

Here we denoted the auxiliary (response) fields as h' and θ'; the needed integrations are omitted for brevity of the notation.

Analysis of canonical dimensions for the model (6) requires additional considerations as the total canonical dimension d_F can be defined in two different ways: either as $d_F^{(2)} = d_F^k + 2d_F^\omega$ following equation (1) or $\omega \sim k^4$ as equation (2) suggests. The definitions correspond to the two different IR asymptotic regimes: $\omega \sim k^2 \to 0$ and $\omega \sim k^4 \to 0$ in the free theory.

The first choice makes couplings with λ_0 and α_0 IR irrelevant (in the sense of Wilson) so it is natural to pick $d_F = d_F^{(4)}$ instead which makes the model (6) nontrivial and logarithmic at $d = 2$.

But now the parameter ξ_0 acquires a negative total dimension; this is why we branded the parameter as IR irrelevant earlier. Dropping the corresponding term from the action functional is the standard procedure, see, e.g. Section 1.16 in [29]. As it turns out, in the present case, doing so is ill-advisedly.

Moreover, to get to the bottom of that issue, minimal subtraction scheme needs to be substituted with a certain non-conventional RG scheme that allows to take that IR irrelevant contribution into account. Indeed, calculations performed in minimal subtraction scheme yield only two types of fixed points of the RG equations: Gaussian fixed point (regime of ordinary difussion) and the point that corresponds to the regime governed solely by the CKPZ equation. However, the latter is a saddle for any value of small parameter $\varepsilon = 2 - d$ and cannot describe the IR asymptotic behaviour of the model.

What is the reason for such abnormality? Consider the linear response function taken at a certain fixed point. The zeroth order approximation for $\langle\theta\theta'\rangle$ in the frequency-momentum representation

$$\langle\theta\theta'\rangle_0 = (-i\omega\xi + k^2)^{-1},$$

explicitly allows to put $\xi = 0$. Such naive approach even yields correct renormalization constants. But the time-coordinate representation

$$\langle\theta\theta'\rangle_0 = \Theta(t)\,(4\pi t)^{-d/2}\,\xi^{d/2-1}\exp\left(-\xi r^2/4t\right), \quad r = |\mathbf{x} - \mathbf{x}'| \qquad (7)$$

is singular when $\xi \to 0$ (here $\Theta(\cdot)$ is the Heaviside step function).

Similar effect was observed in the analysis of the H model of critical dynamics (see Sections 5.24 and 5.25 in [29]) where keeping an IR irrelevant term allows to establish certain exact relations between static and dynamical Green's functions. In [29], Section 1.16, and [50], Section 1.4, and [49], an irrelevant contribution termed "dangerous" signals crucial symmetry violation.

Taking all of the above into account, we pass to a non-conventional approach where we keep the formally irrelevant term $\theta'\partial_t\theta$ following examples set by [54, 55] and use a scheme based on the subtraction at the normalization point at fixed d dimensions [51]. We use the advanced version of this so-called "real-space RG" suggested in [52, 53] where the normalization point is chosen at $p = 0$, $\mu = m$, while the subtraction scheme provides the renormalization constants and the RG functions independent of μ at fixed bare parameters so that the RG equations can be derived in the standard way [29].

The action functional for the problem (2)–(4) reads:

$$S(\Phi) = -\frac{h'\partial^2 h'}{2} + h'\left[-\partial_t h - \partial^2\left(\nu_0\partial^2 h + \frac{1}{2}\lambda_0(\partial h)^2\right)\right] \\ -\theta'\partial_t\theta + \kappa_0\theta'\partial^2\theta - \alpha_0\kappa_0(\partial\theta')(\partial h)\theta. \tag{8}$$

The combination $u = \nu\, m^2/\kappa$ is completely dimensionless and serves as an additional coupling constant. Thus, now we have an extended set of three charges: $g = \alpha\kappa^{-1}\nu^{-1/2}$, $w = \lambda\nu^{-3/2}$ and u. We have calculated the corresponding renormalization constants and RG functions (the β functions and the anomalous dimensions γ) in the leading one-loop approximation. As a result, a new nontrivial fixed point with the coordinates $g^* \neq 0$, $w^* = -\sqrt{4d(2-d)/3}$, $u^* \neq 0$, IR attractive for any $d < 2$, was found. It corresponds to a fully nontrivial IR asymptotic scaling regime for the Green's functions, in which all the nonlinearities of the original stochastic equations are simultaneously relevant.

Existence of one more fixed point, also IR attractive for $d < 2$, looks plausible, but should be considered with a special care because it corresponds to a rather sophisticated asymptotic behaviour $g^* \sim (u^*)^\alpha \to \infty$ with $0 < \alpha \leqslant 1/2$. These points with $u^* \neq 0$ were inevitably overlooked in the standard approach which can be formally reproduced only as the limit $u \to 0$.

Then for the linear response function in the t–$\mathbf{x}$ representation and in the IR range we obtain the following scaling representation:

$$\langle\theta(t,\mathbf{x})\theta'(0,\mathbf{0})\rangle \simeq r^{-d} F\left(t\kappa/r^{2-\gamma_\kappa^*};\, t\nu/r^{4-\gamma_\nu^*}\right), \quad r=|\mathbf{x}| \tag{9}$$

with a certain scaling function F. Here γ_κ^* and γ_ν^* are the anomalous dimensions calculated at the given fixed point.

The mean-square displacement $R(t)$ of a random walker on a rough surface is given by the expression:

$$R^2(t) = \int d\mathbf{x}\, x^2\, \langle\theta(t,\mathbf{x})\theta'(t',\mathbf{x}')\rangle.$$

Thus, there are two possible equivalent scaling representations

$$R^2(t) \sim (\kappa t)^{2/\Delta_\kappa}\, H(z) \sim (\nu t)^{2/\Delta_\nu}\, \widetilde{H}(z), \quad z = (\kappa t)^{\Delta_\nu}/(\nu t)^{\Delta_\kappa},$$

with certain scaling functions $H(z)$ and $\widetilde{H}(z)$.

Existence of two different critical dimensions of time (or frequency), $\Delta_\kappa = 2-\gamma_\kappa$ and $\Delta_\nu = 4-\gamma_\nu^*$, and two corresponding dispersion laws, $\omega\sim\kappa k^2$ and $\omega\sim\nu k^4$, is sometimes referred to as weak scaling [56].

Let us assume that the Green's function (9) in the limits $r\to\infty$ and $t\to\infty$ remains finite and becomes independent on the argument that tends to zero or infinity. For the the case of fixed t/r^{Δ_κ} this assumption is supported by the explicit leading-order approximation (7). Then the expression (9) is reduced either to

$$\langle\theta\theta'\rangle \simeq r^{-d}\widetilde{F}\left(t\kappa/r^{2-\gamma_\kappa^*}\right)$$

(for $r\to\infty$, $t\to\infty$ and fixed t/r^{Δ_κ}), or to

$$\langle\theta\theta'\rangle \simeq r^{-d}\widetilde{F}\left(t\nu/r^{4-\gamma_\nu^*}\right)$$

(for $r\to\infty$, $t\to\infty$ and fixed t/r^{Δ_ν}), where $\widetilde{F}$ are certain scaling functions.

This gives the final power-like asymptotic expressions for the mean-square displacement:

$$R^2(t)\sim(\kappa t)^{2/\Delta_\kappa} \quad \text{and} \quad R^2(t)\sim(\nu t)^{2/\Delta_\nu} \tag{10}$$

with the exponents

$$\Delta_\kappa = \frac{(d+4)}{3}, \quad \Delta_\nu = \frac{(d+10)}{3}\,, \tag{11}$$

calculated in the leading order for the fully non-trivial regime. These expressions becomes exact for $d=2$.

§3. Conclusion

Preliminary results of our study of the random walk, modelled by the Fokker–Planck equation (1), on a random rough surface described by the CRPZ equation (2), (3) revealed that certain formally IR irrelevant parameter (and corresponding contribution to the action functional) cannot be dropped from the problem, otherwise the single nontrivial IR attractive fixed point of the RG equations and the corresponding scaling regime of IR behaviour become undetectable. To take that term into account, it is necessary to employ a nonconventional RG scheme in the real d-dimensional space.

Established scaling behaviour of the Green's functions (9) involves two different values of time/frequency critical dimension, the phenomenon referred to as weak scaling.

As a result, we obtained two different expressions (10) for the mean-square displacement of a random walker on a rough surface depending on the specific asymptotic sub-domain, with the two different exponents (11).

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