85 anniversary of Professor

Vsevolod Alekseevich Solonnikov

Professor Vsevolod Alekseevich Solonnikov was born on June 8, 1933 in Leningrad. The scientific activity V. A. Solonnikov began in the late 1950s, when he graduated from the Leningrad University with honors degree. He defended the Diploma Thesis in 1957 under the supervision of Professor O. A. Ladyzhenskaya. After that, V. A. Solonnikov was invited to join her research group at LOMI (Leningrad Branch of Steklov Mathematical Institute). Beginning with the first published paper, Vsevolod Alekseevich has obtained new interesting results in various fields related to partial differential equations and theory of functions. Now many of his results are considered as classical. In the PhD Thesis (defended in 1961), V. A. Solonnikov proved L^p estimates for parabolic equations and for the stationary Navier–Stokes system. Later (in the Doctoral Thesis) he developed the theory of initial boundary value problems for general parabolic systems and also obtained analogous results for the non-stationary Stokes system. Professor V. A. Solonnikov has published 280 papers. In this note we briefly describe his main results.

Theory of Functions

A considerable part of the research made by V. A. Solonnikov was devoted to studying functional spaces, proving interpolation and multiplicative inequalities and trace theorems. These type results are contained in many of his papers. Many of them were obtained under the influence of professor V. P. Il'jin, who may be considered as a teacher of Vsevolod Alekseevich in the area related to function theory.

In the 60s, Professor Solonnikov suggested (in his joint paper with K. K. Golovkin) conditions under which the convolution operator is bounded for a wide class of fractional spaces. They proved theorems on the Fourier multipliers in Hölder and Besov spaces, which enable them to obtain precise estimates for solutions of some PDE's problems in these spaces. In particular, V. A. Solonnikov used this technique to obtain the exact estimates for a solution to a non-coercive problem arising from free boundary problems for the Navier–Stokes equations.

Elliptic and Parabolic Problems

In the early 60s, V. A. Solonnikov deduced coercive estimates of solutions to boundary value problems for general elliptic systems (so called ADN-elliptic systems). These results were obtained by Vsevolod Alekseevich independently[1], however they have been published a little bit later than in the famous paper:

S. Agmon, A. Douglis, L. Nirenberg, *Estimates near the boundary for solutions of elliptic partial differential equations satisfying general boundary conditions, II*, *Comm. Pure App. Math.*, (1964).

At the same time professor Solonnikov developed the solvability theory in Sobolev and Hölder spaces[2] for initial boundary value problems generated by parabolic system of a general form. In order to solve this problem, he proposed a new method including the construction of a regularizer. Next, he constructed Green's matrices for elliptic boundary value problems which gave explicit solutions.

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[1] V. A. Solonnikov, *On general boundary value problems for A. Douglis–L. Nirenberg elliptic systems, I*, *Izvestia Acad. Sci. USSR ser. matem.*, **28**, 3 (1964), 665–706.

V. A. Solonnikov, *On general boundary value problems for A. Douglis–L. Nirenberg elliptic systems, II*, *Trudy Mat. Inst. Steklov*, **92**, (1966), 233–297.

[2] V. A. Solonnikov, *On boundary value problems for linear parabolic systems of differential equations of general form*, *Trudy Mat. Inst. Steklov*, **83**, (1965), 3–162.

Describing the results of professor Solonnikov related to elliptic and parabolic problems, it is necessary to mention the famous book

O. A. Ladyzhenskaya, V. A. Solonnikov and N. N. Uraltseva, *Linear and Quasilinear Equations of Parabolic Type*. Moscow, "Nauka", 1967; Engl. trans.: *Translations of Mathematical Monographs*, **23**. *Providence, R.I.: American Mathematical Society*, 1968.

In the 80s, there appeared several papers by V. A. Solonnikov where he proved the solvability of boundary value problems for domains with corners and edges at the boundary. These results were very important for the investigation of free boundary problems associated with the Navier–Stokes equations and enriched the theory of PDE's in domains with singular boundaries developed by V. A. Kondrat'ev, V. G. Maz'ya and B. A. Plamenevskii.

By analyzing free boundary problems for parabolic equations, V. A. Solonnikov has investigated non–coercive problems with dynamic boundary conditions in various functional spaces. For example, he has proved a local (in time) classical solvability for the Stefan and Verigin problems with minimal order compatibility conditions on the boundary. The solutions found had at least the same smoothness as at the initial time moment. These results were obtained by Vsevolod Alekseevich jointly with his pupils G. I. Bizhanova and E. V. Frolova.

Incompressible Navier–Stokes Equations

Using his estimates of integral operators, V. A. Solonnikov obtained coercive estimates for solutions of the initial boundary value problem for the nonstationary Stokes equations. Based on these results, he later investigated differential properties of general solutions for the nonstationary Navier–Stokes problem.

Since the beginning of 1970s, scientific interests of V. A. Solonnikov were mainly focused on fluid dynamics. In 1973, he presented another proof of the coercivity estimates (extremely exact) for the nonstationary Stokes problem in Sobolev and Hölder spaces which nowadays are considered as classical. Later on he had also derived similar estimates in $L^{q,r}$-spaces.

In 1976, he started to investigate boundary value problems governing fluid motion in domains with noncompact boundaries. In the fundamental

papers by O. A. Ladyzhenskaya and V. A. Solonnikov[3], the right choice of functional spaces was made, which made it possible to prove existence theorems in the case, where "outlets" to infinity contain cones (and later in the case of arbitrary "outlets" to infinity). The case where the outlets are "sufficiently wide" was studied by Vsevolod Alekseevich in collaboration with his pupil K. Pileckas. The obtained results and the new methods developed opened a way to investigate the Navier–Stokes equations in non-compact domains. This topic is very popular in the mathematical community and far from to be complete until now. It is worth to say that the research of many mathematicians all over the world is based on these fundamental ideas of V. A. Solonnikov.

Professor Solonnikov obtained global existence results for many nonstationary free boundary problems. Also, he considered the thermocapillary convection problem and proved global solvability of this evolutional problem for incompressible liquid.

In the 80s, V. A. Solonnikov started to study the drop motion problem in a general setting. In 1984, he posed the problem on evolution of an isolated volume of viscous incompressible fluid and gave a detailed scheme of proving the local unique solvability of it in anisotropic Sobolev–Slobodetskii spaces. However, the problem turned out to be much more complicated than it was expected at the beginning. To prove the solvability for the corresponding linear problem, V. A. Solonnikov has used his method of constructing a regulariser. The complete proof of the local existence theorem for the problem of motion of a viscous drop was finished only in the early 90s.

In Hölder spaces, a similar result for this problem was established by V. A. Solonnikov in collaboration with his another pupil I. Sh. Mogilevskii. They apply a rather complicated method of Fourier multipliers for Hölder spaces. Later on, professor Solonnikov replaced this complicated technique by a simpler one using Hölder spaces with reduced smoothness in time.

[3]O. A. Ladyzhenskaya, V. A. Solonnikov, *On some problems of vector analysis and on generalized formulations of boundary value problems for the Navier–Stokes equations*, *Zapiski Nauchn. Semin. LOMI*, **59**, (1976), 81–116.

O. A. Ladyzhenskaya, V. A. Solonnikov, *Determination of the solutions of boundary value problem for stationary Stokes and Navier–Stokes equations having an unbounded Dirichlet integral*, *Zapiski Nauchn. Semin. LOMI*, **96**, (1980), 117–160.

V. A. Solonnikov, *Solvability of boundary and initial-boundary value problems for the Navier–Stokes equations in domains with non-compact boundaries*, *Zapiski Nauchn. Semin. LOMI*, **96**, (1980), 288–293.

Next, he together with his pupil I. V. Denisova developed this simplified method and proved the local solvability of the problem of bubble motion without loss of regularity with respect to time. In the case of small initial velocities and in the case where the initial shape of the drop is close to a ball, professor Solonnikov showed that the problem of motion of an isolated volume of a self-gravitating capillary fluid is solvable in an infinite time interval.

Stability of equilibrium figures of rotating fluids

In the early 2000s, Professor Solonnikov started to study the stability of equilibrium figures of rotating fluids. The figures of equilibrium of rotating homogeneous self-gravitating liquids and their stability has been of interest for mathematicians over several centuries. Many famous scientists such as Newton, Maclaurin, Jacobi, Liouville, Poincaré, Lyapunov and others have made their contributions in analysis of this problem. In papers of V. A. Solonnikov, the stability question was related to the question of the sign of the second variation of the energy functional. The results of Vsevolod Alekseevich on stability of axially symmetric and nonsymmetric equilibrium figures (subjected or not subjected to capillary forces on the boundary) may be regarded as justification of classical Lyapunov's conclusions concerning the stability of equilibrium figures of a rotating fluid in the absence of surface tension. Some of these results V.A. Solonnikov obtained jointly with Professor M. Padula during his stay in the University of Ferrara (Italy)[4]. We can say that these recent results of V. A. Solonnikov constitute a complete mathematical theory of the stability of rotating viscous incompressible fluids with free boundaries.

Magnetohydrodynamics

V. A. Solonnikov started his scientific activity with the study of magnetic hydrodynamics problems. At that time, Professor O. A. Ladyzhenskaya was his superviser. She has established the global solvability of 2D Navier–Stokes equations and the solvability for small data (or small time interval) for 3D Navier–Stokes equations (jointly with A. A. Kiselev). She proposed Vsevolod Alekseevich to obtain similar results for the magnetohydrodynamics system. As a result, they proved local unique solvability

[4] M. Padula, V. A. Solonnikov, *A simple proof of the linear instability of rotating liquid drops*, *Ann. Univ. Ferrara Sez. VII Sci. Mat.* **54**, 1, (2008), 107–122.

of unsteady magnetic hydrodynamics problem in $\mathbb{R}^3$ and global (in time) unique solvability of two-dimensional problem[5]. In his own paper, Vsevolod Alekseevich considered the stationary problem[6]. After many years, professors O. A. Ladyzhenskaya and V. A. Solonnikov returned to magnetohydrodynamics by considering the stability and instability of stationary and periodic solutions[7].

In the period of 2010–2018, Vsevolod Alekseevich again returned to MHD problems. He studied problems with free boundaries in multi-connected domains. Such problems have many applications (for example, TOKAMAK). Some of these results were obtained jointly with M. Padula[8] during the stay of V. A. Solonnikov in Ferrara university.

Some other recent results of V. A. Solonnikov

In the beginning of 2000 the group in POMI (Ladyzhenskaya, Seregin, Shilkin, etc.) were intensively studying modified Navier–Stokes equations (so called "Ladyzhenskaya system"). It was necessary to study the linearized problem with the main part different from Laplacian. The corresponding theory of coercive estimates for such system was developed by V. A. Solonnikov. Simultaneously, he obtained also a series of new results for the classical Stokes system. Widely known are results concerning the problem with initial conditions which do not vanish at infinity.

The scientific achievements of V. A. Solonnikov were highly appreciated by the international mathematical community: he became professor of Ferrara University (Italy) and a foreign member of the Lisbon Academy of Sciences. To celebrate his 70th, 75th, and 80th jubilees international conferences were organized in Germany, Portugal, Italy, Poland and Russia. The international conference "The last 60 years of Mathematical Fluid Mechanics: Longstanding Problems and New Perspectives. In Honor of

[5]O. A. Ladyzhenskaya, V. A. Solonnikov, *Solvability of evolution problems of magnetohydrodynamics*, *Doklady Acad. Sci. USSR*, **124**, 1 (1959), 26–28.

[6]V. A. Solonnikov, *On some stationary problems for equations of magnetohydrodynamics of a viscous incompressible liquid*, *Proc. V All-union conference on functional analysis*. Baku (1961), 241–246.

[7]O. A. Ladyzhenskaya, V. A. Solonnikov, *The linearization principle and invariant manifolds for problems of magnetohydrodynamics*, *Zapiski Nauchn. Semin. LOMI*, **38**, (1973), 46–93.

[8] M. Padula, V. A. Solonnikov, *On the free boundary problem of magnetohydrodynamics*, *Zap. Nauchn. Semin. POMI*, **385** (2010).

Professors Robert Finn and Vsevolod Solonnikov" was organized in 2017 in Vilnius.

Professor Solonnikov has been visiting professor in many academic institutions, including University of Paderborn (Germany), University of Rome (Italy), University of Lisbon (Portugal), University of Zurich (Switzerland), Waseda University (Japan), Keio University (Japan), University of Toulon (France), University of Minnesota (USA), University of Catania (Italy) and many others.

He was awarded the Humboldt prize (2003); the M. A. Lavrentjev prize of the Russian Academy of Sciences (2009), the L. B. Chebyshev award of the government of St. Petersburg 2013. In 2015, he was awarded the title "Honored worker of science of the Russian Federation". In 1992–1996, he was a member of the Executive Commettee of the European Mathematical Society.

He is (was) a member of the Editorial Board of several international scientific Journals, including SIAM Journal of Mathematical Analysis, Interfaces and Free Boundaries, Journal of Mathematical Fluid Mechanics, Annali dell'Universita di Ferrara, Advances in Mathematical Sciences and Applications, Differential Equations and others.

As of nowadays, Professor V. A. Solonnikov is still scientifically very active and, as a broad-minded person, always ready to discuss and give his precious insights to most complicated and important mathematical problems. The pupils of Vsevolod Alekseevich will always be thankful to him not only for knowledge that he passed to them but also for warmheartiness and interest he showed in their research and life. They also felt the great hospitality of his wife Tatyana Fedorovna, unfortunately, he passed away in October, 2002.

On behalf of his friends, pupils, and colleagues I offer to Vsevolod Alekseevich our heartiest congratulations and wish him health, happiness and many years of successful scientific work, as a natural continuation of his fundamental work over the past sixty-plus years!

Konstantin Pileckas