Записки научных
семинаров ПОМИ
Том 437, 2015 г.

V. Arzumanian, S. Grigoryan

GROUP-GRADED SYSTEMS AND ALGEBRAS

Abstract. In the paper, we discuss some problems concerning the structural properties of crossed products. While expansions of C^*-algebras under group actions have been studied rather extensively, known difficulties in the transition to irreversible dynamical systems require the development of new methods. The first step in this direction is to study actions of inverse semigroups, whose properties are closest to those of groups. The main tool to achieve the goal is the concept of grading. The detection of the grading structure in the corresponding constructions seems to be very promising.

Introduction

The initial motivation to present these notes was an attempt to find a satisfactory construction of the operator algebra associated with a semigroup dynamical system, a theme that has become very popular in recent years. Well-known difficulties arising in the irreversible case due to the lack of complete analogy with group systems gave rise to many constructions, none of which is entirely acceptable.

Analyzing the numerous studies on the subject, one should conclude that the idea of grading in algebras plays a central role in most constructions, although is not always explicitly present. Moreover, the concepts of crossed product and graded systems seem to be indiscernible.

When these notes were ready for publication, the authors discovered that these ideas have been already implemented to some extent.

We are grateful to Dr. Marat Aukhadiev who, after having read our research, sent us a link first to the article [1] by Buss and Exel, and then to the remarkable monograph [2] by Ruy Exel, in which the above-mentioned program is developed methodically and thoroughly.

We cannot resist the temptation to quote a phrase from this work, in which the basic idea of the program lies: " ... graded algebra satisfying quite general hypotheses is necessarily a partial crossed product!"

Key words and phrases: C^*-algebra, representation, conditional expectation, bimodule, Hilbert module, graded system, graded C^*-algebra, inverse semigroup.

In addition, it was found that a similar understanding, apparently, was long ago expressed by A. Kumjian in [3].

Although some of the proposed approaches turned out to be already known (the main of which is the delimitation between the concepts of graded system and graded algebra), the objectives that we set ourselves are different from those explored in the above-mentioned work by Exel.

There are other differences, too. We do not use partial actions and partial crossed products, the main leitmotif of Exel's work. We believe that in studying semigroup dynamical systems one can reach the goal applying the action of a suitable group on a modified initial algebra with complementary relations, which should accumulate all difficulties related to partial isometries.

In the first section, we consider the concept of a group-graded system (which is apparently the Fell bundle mentioned in the papers cited above) and accompanying notions. We do not change terms and notation: the relationship between our vision and the concepts from Exel's work is the subject of a detailed analysis in the nearest future. We also consider the representations in the associated Hilbert module and show that a graded system can be implemented as an operator system under the action of the regular representation.

In the second section, we introduce the concept of a graded C^*-algebra and study the relations between group-graded systems and graded C^*-algebras. Note that our definition differs from that used in the above-mentioned book: what we call a graded algebra is a topological grading there.

In the last section, we introduce the notion of a coupling of a C^*-algebra and an inverse semigroup, with the goal of constructing associated graded systems and algebras.

This paper should be seen as a brief introduction into the related issues.

§1. Group-graded systems

1.1. Definitions and elementary properties. Let $A_\Gamma = \{A_\gamma,\, \gamma \in \Gamma\}$ be a collection of Banach spaces A_γ (with norm $\|\cdot\|_\gamma$ and zero θ_γ) indexed by a discrete group Γ (with neutral element ϵ) such that $A = A_\epsilon$ is a C^*-algebra (called *central*) with unit $\mathbf{1}$, zero element $\mathbf{0}$, and norm $\|\cdot\|_\epsilon := \|\cdot\|$.

Let us also agree that $\xi \in A_\Gamma$ means $\xi = \{\xi_\gamma\}_{\gamma\in\Gamma}$.

Definition 1. *A system $\mathcal{A} = (\Gamma, A_\Gamma, \sigma, \tau)$ is called a* Γ-graded *system if there exist systems of mappings $\sigma = \{\sigma_\gamma\}_{\gamma\in\Gamma}$ and $\tau = \{\tau_{\gamma,\gamma'}\}_{\gamma,\gamma'\in\Gamma}$*

satisfying the following conditions I–IV (*we always omit the indices of mappings, norms, etc. without loss of meaning, since arguments identify them unambiguously*).

I. *A bilinear operator* $\tau : A_\alpha \times A_\beta \to A_{\alpha\beta}$ *is defined on each pair of spaces and satisfies the conditions*

$$\tau(\tau(a,b),c) = \tau(a,\tau(b,c)) \quad \text{for} \quad a \in A_\alpha,\ b \in A_\beta,\ c \in A_\gamma, \tag{1}$$

$$\tau(a,b) = ab \quad \text{for} \quad a,b \in A, \tag{2}$$

$$\tau(a,\mathbf{1}) = \tau(\mathbf{1},a) = a \quad \text{for} \quad a \in A_\alpha. \tag{3}$$

II. *An antilinear bijection* $\sigma : A_\gamma \to A_{\gamma^{-1}}$ *is defined on each space and satisfies the conditions*

$$\sigma^2(a) = a \quad \text{for} \quad a \in A_\alpha, \tag{4}$$

$$\sigma(a) = a^* \quad \text{for} \quad a \in A. \tag{5}$$

III. *The partial mappings* τ *and* σ *satisfy the consistency conditions*

$$\sigma(\tau(a,b)) = \tau(\sigma(b),\sigma(a)) \quad \text{for} \quad a \in A_\alpha, b \in A_\beta, \tag{6}$$

$$\tau(\sigma(a),a),\ \tau(a,\sigma(a)) \in A_+ \quad \text{for} \quad a \in A_\alpha, \tag{7}$$

and

$$\tau(\sigma(a),a) = \mathbf{0}, \quad a \in A_\gamma, \quad \text{if and only if} \quad a = \theta_\gamma. \tag{8}$$

IV. *The following* C^**-property is satisfied for every* $a \in A_\gamma$:

$$\|\tau(a,\sigma(a))\| = \|\tau(\sigma(a),a)\| = \|a\|_\gamma^2. \tag{9}$$

It follows immediately from the definition that

$$\tau(\tau(c,a),b)) = c\tau(a,b), \quad a \in A_\alpha, b \in A_{\alpha^{-1}}, c \in A,$$
$$\tau(\sigma(a),a),\ \tau(a,\sigma(a)) \in A_+, \qquad a \in A_\alpha,$$
$$\|\sigma(a)\|_{\alpha^{-1}} = \|a\|_\alpha, \qquad a \in A_\alpha,$$
$$\|\tau(a,b)\|_{\alpha\beta} \leqslant \|a\|_\alpha \|b\|_\beta, \qquad a \in A_\alpha,\ b \in A_\beta.$$

The concepts of a Γ-graded subsystem, ideal, quotient of a Γ-graded system can be defined in an obvious way.

By a morphism of a graded system $(\Gamma_1, A_{\Gamma_1}, \sigma_1, \tau_1)$ into another graded system $(\Gamma_2, \{B_{\Gamma_2}\}, \sigma_2, \tau_2)$ we understand a pair $\Phi = (\rho, \varphi)$, $\varphi = \{\varphi_\gamma\}_{\gamma\in\Gamma_1}$, such that

(i) $\rho : \Gamma_1 \to \Gamma_2$ is a group homomorphism,

(ii) $\varphi_\gamma : A_\gamma \to B_{\rho(\gamma)}$ is a linear mapping,

(iii) $\sigma_2 \circ \varphi = \varphi \circ \sigma_1$,

(iv) $\tau_2(\varphi_\alpha(a), \varphi_\beta(b)) = \varphi_{\alpha\beta}(\tau_1(a,b))$ for $a \in A_\alpha$, $b \in A_\beta$.

Two graded systems are *isomorphic* if all mappings ρ, φ_γ are bijective.

If $\mathcal{A}$ is a Γ-graded system with an Abelian group Γ, then the *standard action* of the dual group on the system $\mathcal{A}$ can be defined.

Proposition 1. *Let $\mathcal{A}$ be a Γ-graded system with an Abelian group Γ. Then*

$$\alpha(g)(a) = g(\gamma)a \tag{10}$$

is a faithful representation of the dual group $G = \hat{\Gamma}$ into the group of automorphisms of $\mathcal{A}$.

Note that each ideal of a Γ-graded system is invariant under the standard action of the dual group.

1.2. Modules. Let $\mathcal{A}$ be a graded system, B be a C^*-algebra. We introduce a notion of a B-module graded system, limiting ourselves to bimodules. Left and right modules can be defined in a similar way.

Definition 2. *Let $\mathcal{A} = (\Gamma, A_\Gamma, \sigma, \tau)$ be a graded system, B be a unital C^*-algebra. We say that the system $\mathcal{A}$ is a B-module if each A_γ is a B-module and the following consistence conditions are satisfied for all $b \in B$, $\xi \in A_\alpha$, $\eta \in A_\beta$:*

(i) $\tau(b \cdot \xi, \eta) = b \cdot \tau(\xi, \eta)$, $\tau(\xi \cdot b, \eta) = \tau(\xi, b \cdot \eta)$,

(ii) $\sigma(b \cdot \xi) = \sigma(\xi) \cdot b^*$, $\sigma(\xi \cdot b) = b^* \cdot \sigma(\xi)$.

Graded systems have a standard module structure.

Proposition 2. *Let $\mathcal{A} = (\Gamma, A_\Gamma, \sigma, \tau)$ be a Γ-graded system with central algebra A. Then each A_γ, $\gamma \in \Gamma$, is an A-module (more precisely, bimodule) with respect to the operations*

$$a \cdot \xi = \tau(a, \xi) \ \textit{and} \ \ \xi \cdot a = \tau(\xi, a), \quad a \in A,\ \xi \in A_\gamma.$$

Moreover, the system $\mathcal{A}$ is an A-module.

The following proposition introduces a structure of a Hilbert A-module on each A_γ, $\gamma \in \Gamma$.

Proposition 3. *The space A_γ with the inner product*

$$\langle \xi, \eta \rangle = \tau(\sigma(\eta), \xi), \quad \xi, \eta \in A_\gamma, \tag{11}$$

is a (right) Hilbert A-module with the norm $\| \cdot \|_\gamma$.

Then a Γ-graded system $\mathcal{A}$ itself is a right Hilbert A-module, which will be called *associated* with the system $\mathcal{A}$ and denoted by $\mathcal{H}(\mathcal{A})$.

Note that the inner product in $\mathcal{H}(\mathcal{A})$ is given by

$$\langle \xi, \eta \rangle = \sum_{\gamma \in \Gamma} \tau(\sigma(\eta_\gamma), \xi_\gamma), \quad \xi, \eta \in A_\Gamma. \tag{12}$$

Recall that a system $\xi \in A_\Gamma$ belongs to $\mathcal{H}(\mathcal{A})$ if the series $\sum_\gamma \tau(\sigma(\xi), \xi)$ is convergent in A. This condition will certainly be satisfied if the series $\sum_\gamma \|\xi\|_\gamma^2$ converges.

Let $|\xi| = (\tau(\sigma(\xi), \xi)^{\frac{1}{2}}$ for $x \in A_\Gamma$. Then $\xi \in \mathcal{H}(\mathcal{A})$ if and only if $\sum_\gamma |\xi|^2$ is convergent. Thus, the Hilbert module associated to a graded system may be regarded as a noncommutative $l^2(\mathcal{A})$.

1.3. Representations. Graded systems can be regarded as something like covariant systems. Thus, the next notion is an analog of that of a covariant representation.

Definition 3. *Let* $\mathcal{A} = (\Gamma, A_\Gamma, \sigma, \tau)$ *be a* Γ*-graded system and* $\mathcal{H}$ *be a Hilbert space. We call a system* $\pi = \{\pi_\gamma\}_{\gamma \in \Gamma}$ *a* representation *of the system* $\mathcal{A}$ *in* $\mathcal{H}$ *if each* $\pi_\gamma : A_\gamma \to \mathcal{B}(\mathcal{H})$ *is a linear mapping and* (*as usual, we omit the indices*)

(i) $\pi(\tau(a, b)) = \pi(a)\pi(b)$ *for* $a \in A_\gamma$, $b \in A_\beta$,

(ii) $\pi(\sigma(a)) = \pi(a)^*$ *for every* $a \in A_\gamma$.

Obviously, π is a representation of the central algebra A.

It is easy to verify that the kernel of any representation of a graded system is an ideal of the graded system.

Proposition 4. *Every representation* π *of a* Γ*-graded system* $\mathcal{A}$ *is continuous, namely, for every* $a \in A_\gamma$,

$$\|\pi(a)\| \leqslant \|a\|_\gamma. \tag{13}$$

A representation π *is faithful if and only if* $\|\pi(a)\| = \|a\|_\gamma$.

The uniformly closed involutive algebra $C^*(\mathcal{A}, \pi)$ generated by all $\pi(A_\gamma)$ in $\mathcal{B}(\mathcal{H})$ will be called the C^*-algebra *associated* to the graded system $\mathcal{A}$ and the representation π.

The *enveloping* Γ-graded system $\widehat{\mathcal{A}}$ is the graded system A_Γ with the same operations and the norms

$$\|a\|_{\widehat{\gamma}} = \sup \|\pi(a)\|,$$

where $a \in A_\gamma$ and π runs over all representations of the system $\mathcal{A}$.

Definition 4. *Let $\mathcal{A} = (\Gamma, A_\Gamma, \sigma, \tau)$ be a Γ-graded system with central algebra A and $\mathcal{H}$ be a Hilbert A-module. We call a system $\Pi = \{\Pi_\gamma\}_{\gamma\in\Gamma}$ an* A-representation *of the system $\mathcal{A}$ in $\mathcal{H}$ if each $\Pi_\gamma : A_\gamma \to \mathcal{L}(\mathcal{H})$ (the algebra of adjointable bounded A-linear operators on $\mathcal{H}$) is a linear mapping and*

(i) $\Pi(\tau(a, b)) = \Pi(a)\Pi(b)$ for $a \in A_\alpha$, $b \in A_\beta$,

(ii) $\Pi(\sigma(a)) = \Pi(a)^*$ for every $a \in A_\alpha$.

The uniformly closed involutive algebra $C^*_A(\mathcal{A}, \Pi)$ generated by all $\Pi(A_\gamma)$ in $\mathcal{L}(\mathcal{H})$, where Π is an A-representation in a Hilbert A-module $\mathcal{H}$, will be called the C^*-algebra *associated* to the graded system $\mathcal{A}$ and the representation Π.

The *enveloping* Γ-graded system $\mathcal{A}^A$ is the graded system A_Γ with the same operations and the norm

$$\|a\|^A_\gamma = \sup \|\pi(a)\|,$$

where $a \in A_\gamma$ and Π runs over all A-representations of the system $\mathcal{A}$.

Now we introduce a canonical A-representation of a graded system $\mathcal{A}$ in the associated Hilbert module $\mathcal{H}(\mathcal{A})$.

Proposition 5. *The mapping defined on generators $a \in A_\alpha$ and $\xi \in A_\gamma$ as*

$$\Pi_r(a)\xi = \tau(a, \xi)$$

determines an A-representation of the system $\mathcal{A}$ in the associated Hilbert A-module $\mathcal{H}(\mathcal{A})$. It is continuous, and $\|\Pi_r(a)\| \leqslant \|a\|_\gamma$, $a \in A_\gamma$.

Definition 5. *The representation Π_r just introduced is called the (left) regular representation of the graded system $\mathcal{A}$.*

Theorem 1. *The regular representation of any graded system is faithful.*

The last result shows that graded systems can be realized as operator systems. Then the norm of the enveloping algebra $\|\cdot\|_{\widehat{\gamma}}$, the norm of the enveloping graded system $\|\cdot\|^A_\gamma$, and the initial norm $\|\cdot\|_\gamma$ coincide on each A_γ.

Let us denote the uniformly closed subalgebra in $\mathcal{L}(\mathcal{H}(\mathcal{A})))$ generated by the image of the regular representation of a graded system $\mathcal{A}$ by $C^*(\mathcal{A})$.

Then a covariant functor from the category of graded systems (with the above-mentioned morphisms) to the category of C^*-algebras with *-homomorphisms as moprphisms is defined.

In the next section, we describe the image of this covariant functor.

§2. Graded $\mathbf{C}^*$-algebras

We develop the theory of representations of graded systems. Then we pass to graded C^*-algebras.

2.1. The definition of a graded $\mathbf{C}^*$-algebra. Let $\mathfrak{A}$ be a unital C*-algebra, $\mathfrak{A}_\Gamma = \{\mathfrak{A}_\gamma\}_{\gamma\in\Gamma}$ be a system of Banach subspaces of $\mathfrak{A}$ indexed by elements of a discrete group Γ with unity ϵ.

We assume the following:

(i) $\mathfrak{A}_{\gamma^{-1}} = \mathfrak{A}_\gamma^*$,

(ii) $\mathfrak{A}_\gamma \mathfrak{A}_{\gamma'} \subset \mathfrak{A}_{\gamma\gamma'}$,

(iii) $\mathfrak{A}_\gamma \cap \mathfrak{A}_{\gamma'} = \{0\}$, $\gamma \neq \gamma'$.

It is easy to see that $(\mathfrak{A}_\Gamma, \Gamma)$ is a Γ-graded system where the composition (τ) and involution (σ) operations coincide with the corresponding operations of $\mathfrak{A}$.

If the linear span of $\mathfrak{A}_\Gamma$ is dense in $\mathfrak{A}$, we say that this system is associated with $\mathfrak{A}$.

Theorem 2. *Let $\mathfrak{A}$ be a unital C*-algebra, $\mathcal{A} = (\mathfrak{A}_\Gamma, \Gamma)$ be an associated Γ-graded system. There exists a surjective $*$-homomorphism from the algebra $C^*(\mathcal{A})$ onto the algebra $\mathfrak{A}$.*

Definition 6. *We say that an algebra $\mathfrak{A}$ is* Γ-graded *if it is generated by the linear combinations of the elements of $\mathfrak{A}_\Gamma$ and there is a faithful conditional expectation $\mathcal{P} : \mathfrak{A} \longrightarrow \mathfrak{A}_\epsilon$ such that $\mathcal{P}(\mathfrak{A}_\gamma) = 0$ for $\gamma \neq \epsilon$.*

Remark 1. If a C^*-algebra $\mathfrak{A}$ is Γ-graded, then for any $\gamma \in \Gamma$ there is a projection $\mathcal{P}_\gamma$ onto $\mathfrak{A}_\gamma$ such that $\mathcal{P}(\mathfrak{A}_{\gamma'}) = 0$ for $\gamma' \neq \gamma$.

2.2. Some properties. The standard action of the dual group on a graded system can be specified in the case of graded C^*-algebras.

Theorem 3. *Let $\mathfrak{A}$ be a Γ-graded C^*-algebra, with Γ being Abelian, and let G be its dual group, $G = \hat{\Gamma}$. Then there exists a strongly continuous homomorphism $\alpha : G \longrightarrow \mathrm{Aut}(\mathfrak{A})$ such that*

$$\mathfrak{A}_\gamma = \{a \in \mathfrak{A} :\ \alpha_g(a) = g(\gamma)a\}.$$

The converse is also true.

Theorem 4. *Let $\mathfrak{A}$ be a unital C^*-algebra, Γ be a discrete Abelian group, and G be its dual group, $G = \hat{\Gamma}$. Let there exist a strongly continuous homomorphism $\alpha : G \longrightarrow \mathrm{Aut}(\mathfrak{A})$, and let $\mathfrak{A}_\gamma = \{a \in \mathfrak{A} :\ \alpha_g(a) = g(\gamma)a\}$. Then $\mathfrak{A}$ is Γ-graded by the system $\{\mathfrak{A}_\gamma\}$.*

An important question is to find a criterion on a representation of a graded system for its C^*-image to be graded too. The following result gives an answer to this question.

Theorem 5. *Let $\mathcal{A}$ be a Γ-graded system, π be a representation of $\mathcal{A}$. Then the associated algebra $C^*(\mathcal{A}, \pi)$ is Γ-graded if and only if for every finite collection $a_i \in A_{\gamma_i}$, $i = 1, 2, \ldots, n$,*

$$\|\pi(a_i)\| \leqslant \|\sum_{j=1}^{n} \pi(a_j)\|. \tag{14}$$

§3. Coupling of a C*-algebra and a semigroup

To any graded semigroup (we omit here the precise definitions and constructions) and a C^*-algebra one can associate an algebra which can be regarded as an expansion of the initial algebra via the semigroup. Here we limit ourselves to inverse semigroups, which automatically have a natural grading.

3.1. Definitions. Let S be an inverse semigroup without zero divisors, I be an idempotent subsemigroup of S, R be the canonical congruence, $\Gamma = S/R$ be the corresponding group (here we do not analyze problems related to the existence of zero and unit in an inverse semigroup).

The quotient class of an element $s \in S$ will be called its *index* and denoted by $\mathrm{ind}(s)$. Obviously, $\mathrm{ind}(st) = \mathrm{ind}(s)\mathrm{ind}(t)$ and $\mathrm{ind}(s^*) = \mathrm{ind}(s)^{-1}$.

Denote by $A(S)$ the free A-module over S, which is obviously an involutive algebra.

The notion of the index in a semigroup S can be obviously exported to the algebra $A(S)$: if a is a word from $A(S)$, delete all letters in a that are from the algebra A and put the index of a equal to the index of the remaining element of S (assuming that $\mathrm{ind}(\varnothing) = \epsilon$, where ϵ is the unity of Γ).

Thus, this algebra can be represented as the union $\sqcup_{\gamma\in\Gamma} A_\gamma$ of the subalgebras corresponding to the elements of $A(S)$ with the same index, that is,

$$A_\gamma = \{a \in A(S), \mathrm{ind}(a) = \gamma\}.$$

Obviously, $A_\gamma \cap A_{\gamma'} = \varnothing$, $\gamma \neq \gamma'$, $A_\gamma^* = A_{\gamma^{-1}}$, $A_\gamma A_{\gamma'} \subset A_{\gamma\gamma'}$, and $A \subset A_\epsilon$ (in the obvious sense, this is a Γ-grading).

We denote the universal C^*-algebra associated with $A(S)$ by $C^*(A(S))$.

Definition 7. *The algebra $C^*(A(S))$ is called the* coupling *of the algebra A and the inverse semigroup S.*

For a $*$-homomorphism ϕ of the algebra $A(S)$ into a unital C^*-algebra $\mathcal{A}(\phi)$ with norm $\|\cdot\|_\phi$, denote by $\mathcal{A}_\gamma(\phi)$ the closure of the image of A_γ, that is, $\mathcal{A}_\gamma(\phi) = \overline{\phi(A_\gamma)}$.

Definition 8. *The triple (A, S, ϕ) is called a ϕ-*covariant *system if the ϕ-image of $A(S)$ is dense in $\mathcal{A}(\phi)$ and there exists a faithful conditional expectation*

$$p : \mathcal{A}(\phi) \to \mathcal{A}_\epsilon(\phi)$$

*such that $p(a) = 0$ for $a \in \mathcal{A}_\gamma(\phi)$, $\gamma \neq \epsilon$. The algebra $\mathcal{A}(\phi)$ is called the ϕ-*extension *of the algebra A via the semigroup S. The algebra A is called ϕ-*closed *if it is isomorphic to its ϕ-extension.*

It is easy to verify also that for each element $\gamma \in \Gamma$ there exists a projection $p_\gamma : \mathcal{A}(\phi) \to \mathcal{A}_\gamma(\phi)$ such that $p_\gamma(a) = 0$ for $a \in A_{\gamma'}$, $\gamma' \neq \gamma$.

3.2. Main properties. First we show that ϕ-extensions always exist.

Since the algebra $A(S)$ is an A-module, we introduce an A-inner product which is defined on generators as

$$\langle a_\gamma, b_\gamma \rangle = b_\gamma^* a_\gamma.$$

Then we can consider the left regular representation π_r of the algebra $A(S)$ in the corresponding Hilbert A-module.

Theorem 6. *The triple (A, S, π_r) is a π_r-covariant system.*

Returning to an arbitrary ϕ-covariant system, we mention the following result.

Theorem 7. *The algebra $\mathcal{A}(\phi)$ of a ϕ-covariant system is Γ-graded.*

Denote by $C^*(A, S)$ the C^*-algebra that is the closure of $A(S)$ in the norm
$$\|a\| = \sup_{\phi} \|\phi(a)\|_{\phi}.$$

Theorem 8. *The algebras $C^*(A(S))$ and $C^*(A, S)$ are isomorphic.*

Then, the coupling algebra can be uniquely reconstructed from ϕ-covariant systems.

Theorem 9. *For any ϕ-covariant system (A, S, ϕ) there exists a surjective $*$-homomorphism $h_\phi : C^*(A, S) \to \mathcal{A}(\phi)$ from the coupling algebra $C^*(A, S)$ into the algebra $\mathcal{A}(\phi)$ such that the following diagram is commutative*:

$$\begin{array}{ccc} A(S) & \xrightarrow{\mathfrak{i}} & C^*(A,S) \\ \phi \searrow & & \swarrow h_\phi \\ & A(\phi) & \end{array}$$

where $\mathfrak{i}$ *is the canonical imbedding.*

References

1. A. Buss, R. Exel, *Fell bundles over inverse semigroups and twisted etale groupoids.* — J. Operator Theory **67** (2012), 153–205.
2. R. Exel, *Partial Dynamical Systems, Fell Bundles and Applications.* `http://mtm.ufsc.br/~exel/papers/pdynsysfellbun.pdf`.
3. A. Kumjian, *Fell bundles over groupoids.* — Proc. Amer. Math. Soc. **126**, No. 4 (1998), 1115–1125.

Поступило 11 октября 2015 г.

Institute of Mathematics,
National Academy of Science of Armenia,
Yerevan, Armenia
E-mail: `vicar@instmath.sci.am`

Kazan State Power Engineering University,
Kazan, Russia