SEMINARS

Monday, January 12, 10:00–12:00

Yu. V. Matiyasevich (St.Petersburg): **History of Hilbert's Tenth Problem**.

Abstract: In 1900 great German mathematician David Hilbert stated his famous 23 Mathematische Probleme. The problem 10 (the only decision problem on the list) is about solvability of diophantine equations.

In this introductory talk I am to outline the history of investigations on this problem which culminated in 1970 by a proof of its undecidability. Also I will mention a number of other results which were obtained by the technique developed for proving undecidability of Hilbert's tenth problem. I'll indicate several still open closely related problems as well.

Monday, January 19, 10:00–12:00

F. Pop (University of Pennsylvania): **Elementary equivalence of finitely generated fields**.

Abstract: I plan to give an overview of the odd and new developments, concerning the problem of recovering of the isomorphism type of a function field from the elementary (first order) theory of that field.

Thursday, January 22, 14:30–16:00

M. Carrizosa (University of Paris): **Small points on an Abelian variety with complex multiplication**.

Abstract: We provide lower bound for the canonical height of a point P on a CM Abelian variety A in terms of the degree of the field, generated by P, over the field generated by the torsion points of A. This result is a generalisation of some previous results on the Lehmer problem; it allows us to prove a few instances of the Zilber–Pink conjecture.

Monday, January 26, 10:00–12:00

B. Poonen (Massachusetts Institute of Technology): **Cohomological obstructions to rational points**.

Abstract: In almost every instance where the nonexistence of rational points on a variety over a number field has been proved, the nonexistence can be explained by either the Brauer–Manin obstruction or a descent obstruction based on a vast generalization of Fermat's method of infinite descent. I will give an introduction to these obstructions, and discuss a 3-fold constructed in 2008 that proves that these obstructions, even taken together, are not always sufficient.

Thursday, January 29, 14:30–16:00

N. Jones (University of Montreal): **Serre curves in one-parameter families**.

Abstract: A Serre curve is an elliptic curve defined over the rational numbers whose torsion subgroup has "as much Galois symmetry as possible." I will sketch the proof of a theorem which says that, in appropriately chosen one-parameter families of elliptic curves, almost all specializations are Serre curves. This is joint work with A. C. Cojocaru and D. Grant.

Monday, February 2, 10:00–12:00

Yu. V. Nesterenko: (University of Moscow): **On solubility of Diophantine equations in p-adic numbers**.

Abstract: We shall discuss the following result. If a system of polynomial equations with integer coefficients is soluble modulo nth power of a prime number p for n larger than a constant explicitly expressible in terms of the degrees and the heights of the polynomials, then that system has a p-adic solution.

Thursday, February 5, 14:30–16:00

J. Flenner (University of California, Berkeley): **Relative decidability in Henselian valued fields**.

Abstract: Although Henselian valued fields have been shown to be decidable in some special cases (most notably the p-adics), there can be no hope for a general decision procedure. However, in characteristic zero we describe a relative decision procedure given an oracle for an associated structure of leading terms.

Monday, February 16, 14:00–16:00

M. A. Vsemirnov (St.Petersburg): **Cantor's polynomials and their generalisations**.

Abstract: The two well-known Cantor polynomials give bijections between the set of pairs of nonnegative integers and the set of nonnegative integers. This construction can be generalised to other dimensions in several ways. Is it possible to describe all such polynomial bijections? The statement of the problem is natural and very simple, but unfortunately little is known about it. In my talk I shall survey the known results and describe some of the ways of attacking the problem.

Thursday, February 19, 14:30–16:00

N. Niedermowwe (University of Oxford): **The circle method and Diophantine equations**.

Abstract: In this talk I will give an introduction to the circle method and its application to Diophantine counting problems. The benefits of smoothly weighted versions of the circle method will be discussed, and in an example we shall determine the density of zeros of quadratic forms with some of the variables restricted to powers of a prime.

Tuesday, February 24, 14:00–16:00

U. Zannier (Scuda Normale Superiore di Pisa): **On the greatest common divisor** $(u-1, v-1)$ **for** S**-units** u, v**, and applications**.

Abstract: It is a joint result with P. Corvaja that if u, v are multiplicatively independent integers having all prime factors in a prescribed finite set S, then the greatest common divisor $(u-1, v-1)$ is bounded by a constant times an arbitrarily small fixed positive power of the $\max(|u|, |v|)$. We shall illustrate this and related statements (also in the context of function fields) and give some applications to a number of arithmetic and geometric problems.

Thursday, February 26, 14:00–16:00

T. Wooley (University of Bristol): **Diophantine problems in function fields: an analytic perspective**.

Abstract: We discuss the solubility of Diophantine equations and inequalities in the polynomial ring $F[t]$, with F (mostly) a finite field. As is well-known, for equations one may apply Lang–Tsen theory to establish the existence of non-trivial solutions as soon as the number

of variables is large enough in terms of the degrees of the forms defining the equations. We will explain how, under suitable conditions, the circle method may be applied to establish the Hasse principle in this setting, and also discuss what can be said concerning Diophantine inequalities over function fields. Some of this work is joint with Yu-Ru Liu (at Waterloo), and some is joint with Craig Spencer (IAS and Kansas State).

Monday, March 2, 14:00–16:00

Yu. F. Blu (University of Bordeaux I): **Galois representations and Runge's method**.

Abstract: We apply the Diophantine method of Runge to Serre's problem on Galois representations (joint work with P. Parent).

Monday, March 9, 14:00–16:00

J. Brüdern (Universität Stuttgart): **Limitations to the equidistribution for Waring's problem and Weyl's sums**.

Abstract: We discuss our recent joint work with Dirk Daemen, in particular, a disproof of Vaughan's conjecture concerning the behaviour of Weyl sums on major arcs. Our method makes use of an interplay between the solutions of Waring's problem and those Weyl sums, which are, in a sense, each others Fourier transforms. Thus there is an "uncertainty principle", to be explored in a very elementary way.

Thursday, March 12, 14:00–16:00

A. N. Skorobogatov (University of London): **The Brauer groups of Kummer surfaces and torsion of elliptic curves**.

Abstract: This is a talk about a joint work in progress with Yuri Zarhin. A theorem of Zarhin and the speaker says that if X is a K3 surface over a field k finitely generated over $\mathbb{Q}$, then $\mathrm{Br}(X)/\mathrm{Br}(k)$ is finite. However, this group is not known for a single K3 surface over a number field. We consider the case when X is the Kummer surface constructed from the product of two elliptic curves over $\mathbb{Q}$, and exhibit infinitely many examples when $\mathrm{Br}(X) = \mathrm{Br}(\mathbb{Q})$. This raises the question of weak approximation on such surfaces, and also the unexpected question of whether the size of $\mathrm{Br}(X)/\mathrm{Br}(\mathbb{Q})$ can take only finitely many values for K3 surfaces X over $\mathbb{Q}$.

Monday, March 16, 14:00–16:00

E. Hrushovski (Hebrew University): **Motivic Poisson summation**.

Abstract: The motivic integration problem generalizes various aspects of integration over locally compact fields of positive characteristic, replacing numbers by more structured objects formed out of algebraic varieties, and allowing an arbitrary base field to replace the finite field. I will report on work with Kazhdan extending the same idea to counting rational points over a global field of positive characteristic. As an application, we obtain relations among certain motivic integrals over distinct local fields, or relating to distinct division algebras over one field. I'll begin with a short survey of the relation between the logic of valued fields and motivic integration, and some of the obstacles to extending this relation to the global setting.

Thursday, March 19, 14:00–16:00

Yu. I. Manin (Bonn): **Mordell–Weil problem for cubic surfaces: in search of a model-theoretic approach**.

Abstract: Mordell–Weil theorem for elliptic curves establishes that over a field K which is finitely generated, say, over $\mathbb{Q}$, such a curve C has a finitely generated abelian group $C(K)$ of rational points. The proof allows also to get an asymptotic formula for the number of points of bounded height. If $C(K)$ is nonempty, C can be embedded as a cubic curve in the projective plane, and finite generation can be restated in terms of projective geometry: all point of $C(K)$ can be obtained from an initial finite subset by drawing secants (and tangents) L through initial already constructed points of C and adding their (K-rational) intersection points with C to the previously constructed subset. In this form, conjecture of finite generation can be extended to higher-dimensional cubic hypersurfaces. The crucial two-dimensional case still resists solution since 1970's. In this talk, I will discuss theoretical approaches and computer experiments motivated by the two-dimensional Mordell–Weil type conjecture, and stress the emerging model-theoretic framework which might provide a key to this elusive problem.

Monday, March 23, 14:00–16:00

T. Schlank (Hebrew University): **On rational points of homogeneous spaces**.

Abstract: The study of rational points on a homogeneous space with a finite geometric stabiliser is closely related to some embeddings prob-

lems. Making use of that connection, we obtain a few results on the Brauer–Manin obstruction, on the etale Brauer–Manin obstruction, and on the zero cycles of degree one.

Thursday, March 26, 14:30–15:30

Prem Prakash Pandey (Madras): **The Catalan conjecture over a number field**.

Abstract: There have been some attempts to consider the Catalan equation over number fields. The progress in this direction is not encouraging at all. We try to look at the problem from Mihailescu's point of view. Although the developments are very immature, we shall be able to prove the Cassels criteria in some particular cases.

Monday, March 30, 14:00–16:00

P. Salberger (Universität Göteborg): **Counting solutions to Diophantine equations**.

Abstract: We present some new results on the asymptotic behaviour of the number of integral solutions of Diophantine eqations in boxes. To obtain such results one usually applies the Hardy–Littlewood circle method. But this does not work well for polynomials of high degree in few variables. One can then use the p-adic determinant method of Heath-Brown instead.

Thursday, April 2, 14:30–15:30

O. Marmon (Universität Göteborg): **A generalization of the Bombieri–Pila determinant method**.

Abstract: The determinant method was introduced by Bombieri and Pila to get uniform estimates to the density of integral points on affine plane curves of a given degree. We discuss how to generalize this method to varieties of higher dimension.

Monday, April 6, 14:00–16:00

D. R. Heath-Brown (University of Oxford): **Zeros of three p-adic quadratic forms**.

Abstract: As a special case of a conjecture of Artin, any three p-adic quadratic forms should have a common zero as soon as the number of variables is at least 13. It is known that Artin's conjecture is false in general, but this case is still open. A method of Birch and Lewis,

corrected and refined by Schuur, establishes the conjecture if the residue class field has cardinality q which is large enough and odd. This talk presents a new approach to this problem, which shows that is suffices to have $q > 8$ and odd.

Tuesday, April 7, 10:00–11:30
Triantaphyllos Xylouris (Universität Bonn & Universität Hannover): **On a theorem of Linnik**.

Abstract: Let a and q be co-prime positive integers. In 1944 Yu. V. Linnik proved that the smallest prime in the arithmetic progression $a + ql$ does exceed Cq^L for some positive constants C and L. Subsequently, a concrete value for L was given and then improved several times; finally, in 1992 Heath-Brown proved that $L = 5.5$ is acceptable. In his paper, Professor Heath-Brown described several small improvement potentials; building on that description, we are able to improve the bound to $L = 5.2$. In this talk we shall give an overview of Heath-Brown's work and our improvements.

Thursday, April 9, 14:00–16:00
M. Stoll (Universität Bayreuth): **Rational points on curves of genus 2**.

Abstract: We shall discuss various aspects of the subject in the title; for instance, experimental data and conjectures suggested by them, or algorithmic approaches to the determination of the set of rational points on a given curve.

Thursday, April 16, 14:30–15:30
C. Salgado (University of Paris): **Comparing the rank of the generic and the special fibres on rational elliptic surfaces**.

Abstract: We prove, for a large class of rational elliptic surfaces, that there are infinitely many fibres with rank at least equal to the generic rank plus two.

Monday, April 20, 14:00–16:00
T. D. Browning (University of Bristol): **Density of rational points on Chatelet surfaces**

Abstract: The Manin conjecture predicts the asymptotic growth rate of rational points on rational surfaces. This talk surveys recent approaches to this conjecture for Chatelet surfaces and culminates in an upper bound of the expected order of magnitude.