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DIFFRACTION BY AN ELASTIC WEDGE WITH STRESS-FREE BOUNDARY: EXISTENCE AND UNIQUENESS

Vladimir Kamotski[a] **and Gilles Lebeau**[b]

[a] St. Petersburg Department of V. A. Steklov Institute of Mathematics
27 Fontanka, St Petersburg 191011, Russia, vladimir@pdmi.ras.ru
[b] Laboratoire J.-A. Dieudonnè, Universitè de Nice - Sophia Antipolis
Parc Valrose 06108 Nice Cedex 02 France, lebeau@math.unice.fr
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Abstract

Applying the spectral function techniques, worked out in [CL99] we prove existence of solution to the problem of plane (and spherical) wave diffraction by an elastic wedge with stress-free boundary conditions. We also formulate radiation conditions, under which the uniqueness holds.

1 Introduction

The problem of the diffraction by an elastic wedge has a significant history. A detail review of the works in this direction is given in [KBBF03] and [CL99] thus we will restrict ourselves with a short comment.

The problem under consideration seems very unlikely to have a solution in the closed form, and up to mid 90s of the 20th century most of the authors were interested in the computational aspects of the problem. Several numerical approaches were worked out (see [Fuj94],[Gau02],[Bud95],[KBBF03] and the references therein). The question of correct statement of the problem was never addressed, until the spectral functions techniques was worked out in [Leb95],[CL99]. Addressing a different problem (an immersed elastic wedge) in [CL99] was made a first significant breakthrough for this type of a problem,– the theorem of existence for the plane wave diffraction problem was proved.

In the present work we address problems of plane and spherical waves diffraction by an elastic wedge with stress-free boundary with the help of the spectral functions techniques. We prove existence of the solution and its uniqueness under formulated in the present work radiation conditions for elastic wedge.

2 Notation and results

2.1 Notation

In this section we introduce the notation to be used throughout the paper and which mostly coincide with the ones of [CL99]. Suppose that a wedge of angle $\varphi \in (0, 2\pi)$

$$\Omega = \{(x, y) = (r\cos\theta, r\sin\theta) \ : 0 < \theta < \varphi\} \tag{1}$$

is filled with elastic medium. We introduce two Cartesian systems (x_1, y_1) and (x_2, y_2) associated with wedge faces (see Figure 1), and associated with these systems polar angles θ_1 and θ_2. We have

$$\partial\Omega = \Gamma_1 \cup \Gamma_2, \tag{2}$$
$$\Gamma_j = \{y_j = 0\} = \{\theta_j = 0\}, \ j = 1, 2. \tag{3}$$

In the sequel we follow the agreement that we omit index of coordinate if and only if it is equal to 1.

We consider linearized equations of elasticity and suppose that displacement $u = (u^1, u^2)$ satisfies dynamic Lame system in the wedge

$$\underline{\mu}\Delta u + (\underline{\lambda} + \underline{\mu})\nabla \mathrm{div} u + \rho\omega^2 u = 0 \text{ in } \Omega \tag{4}$$

and stress-free boundary conditions on its faces:

$$(\underline{\lambda}\mathrm{div} u + 2\underline{\mu}\varepsilon(u))n = 0. \tag{5}$$

Here $\underline{\lambda}$, $\underline{\mu}$ are Lame constants, ρ is the density of the medium, ω is the frequency of the process and ε is the deformations tensor given in arbitrary Cartesian coordinates (z_1, z_2) by $\varepsilon_{i,j} = (u^i_{z_j} + u^j_{z_i})/2$.

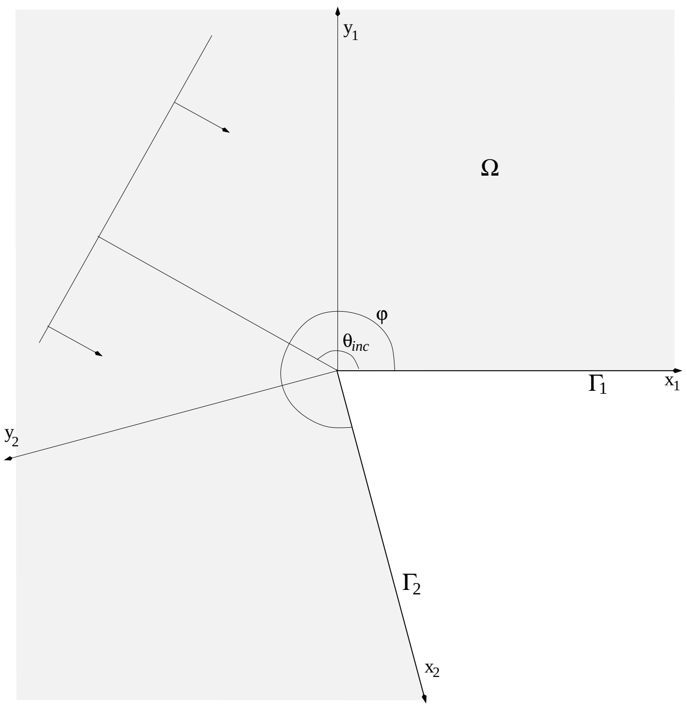

Figure 1: *Geometry of the problem*

As usual we suppose that the total field u is the sum of an incident and scattered parts :

$$u = u^{inc} + u_0, \tag{6}$$

u^{inc} is an incident longitudinal or transversal plane wave.

We change variables to dimensionless: we introduce function v defined by

$$u_0(x, y) = v(\frac{\omega}{c_L}x, \frac{\omega}{c_L}y), \tag{7}$$

where $c_L = \sqrt{(\underline{\lambda} + 2\underline{\mu})/\rho}$ is the longitudinal velocity. Then equations (4)-(6) are equivalent to the system

$$(S^*) \qquad \left\{ \begin{array}{l} (E+1)v = 0 \text{ in } \Omega \\ Bv = -Bv_*^{inc} \text{ on } \Gamma. \end{array} \right.$$

Cases $* = L, T$ correspond to problems of longitudinal and transversal wave diffraction. Here the dimensionless elasticity operator is $E = \mu\Delta u + (\lambda + \mu)\nabla \mathrm{div} u$ and normal stress operator is $Bv = (\lambda \mathrm{div} u + 2\mu\varepsilon(u))n$, with n being an inward normal. The dimensionless incident wave is given by

$$v_L^{inc} = \begin{pmatrix} \cos\theta_{inc} \\ \sin\theta_{inc} \end{pmatrix} e^{ir\cos(\theta - \theta_{inc})} \tag{8}$$

for the case of longitudinal wave incidence, and by

$$v_T^{inc} = \begin{pmatrix} -\sin\theta_{inc} \\ \cos\theta_{inc} \end{pmatrix} e^{i\nu_T r\cos(\theta - \theta_{inc})} \tag{9}$$

for the transversal wave. We have the following relations of dimensionless values with physical ones:

$$\mu = \frac{\mu}{\rho c_L^2},\ \lambda = \frac{\lambda}{\rho c_L^2}, \tag{10}$$

$$\nu_L = 1,\ \nu_T = \frac{c_L}{c_T},\ \nu_R = \frac{c_L}{c_R}, \tag{11}$$

where c_R is the Rayleigh wave velocity. Since $c_T = \sqrt{\underline{\mu}/\rho}$, we have as well $\mu = \nu_T^{-2},\ \lambda+2\mu = 1$.

2.2 Ideology and structure of the paper

Usually when dealing with a diffraction problem, in order to obtain well-posedness, one has to suppose that unknown function satisfies not only the motion equations, but some radiation conditions as well. These conditions are well known for scalar problems of diffraction both for finite and wedge-like scatterers (see e.g. [CH62]).

Following the ideology of [CL99] to select the proper "outgoing" solution of (S^*) we suppose it to satisfy the limiting absorption principle. We consider the problem

$$(S^*_\varepsilon) \qquad \begin{cases} (E + e^{-2i\varepsilon})v = 0 \text{ in } \Omega \\ Bv = -Bv_*^{inc} \text{ on } \Gamma \end{cases}$$

It possesses a unique solution, which as we will demonstrate, can be represented as the sum of two "single layer" potentials, corresponding to two faces:

$$v^\varepsilon = v_1^\varepsilon + v_2^\varepsilon, \tag{12}$$

where functions v_j^ε are defined in entire $\mathbb{R}^2$ by

$$v_j^\varepsilon = -(E + e^{-2i\varepsilon})^{-1} \left[\begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix} \otimes \delta_j \right], \tag{13}$$

here δ_j is an integration measure along the face Γ_j, defined for arbitrary $\psi \in C_0^\infty(\mathbb{R}^2)$ by

$$< \delta_j, \psi >= \int_0^\infty \psi(x_j, 0)\, dx_j. \tag{14}$$

Densities α_j and β_j in (13) are supposed to belong to a special class $\mathcal{A}$ of distributions supported on the positive semi-axis

Definition 2.1. *We say $f \in \mathcal{A}$ if $f \in \mathcal{S}'$, supp$f \subset \mathbb{R}^+$ and if*

$$\exists C_0 > 0 \text{ such that } \sup_{-\pi<\theta<0} \int_{C_0}^\infty |\hat{f}(\rho e^{i\theta})|^2\, d\rho < \infty, \tag{15}$$

$$\hat{f} \text{ is holomorphic in neighbourhoods of } \nu_T,\ \nu_L, \text{ and } \nu_R. \tag{16}$$

Here we used the notation $\hat{f}(\xi)$ for the Fourier transform of the function f:

$$\hat{f}(\xi) = \int_{\mathbb{R}} e^{-ix\xi} f(x)\, dx. \tag{17}$$

We have the following regularity result for potentials v_j^ε (see [CL99], Lemma 2.2)

Lemma 2.2. *Assume $\alpha_j, \beta_j \in \mathcal{A},\ j = 1,2$. Then the two tempered distributions v_j^ε defined by* (13) *converge when $\varepsilon \to 0$ to two tempered distributions of $\mathcal{S}'(\mathbb{R}^2)$ v_j^0 being the solutions of*

$$(E+1)v_j^0 = -\begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix} \otimes \delta_j, \tag{18}$$

moreover the following regularity holds for any $\varepsilon \in [0,\pi)$:

(i) v_j^ε are continuous in $\mathbb{R}^2$ and the traces $v_j^\varepsilon|_\Gamma \in H^1_{loc}(\Gamma)$,

(ii) the traces $\partial_n^\pm v_j^\varepsilon \in L^2_{loc}(\Gamma)$ and are tempered distributions in the neighbourhood of ∞.

Definition 2.3. *We call v an* **outgoing solution** *of (S^*) a solution of the form*

$$v = v_1 + v_2, \tag{19}$$

with

$$v_j = -\lim_{\varepsilon \to 0} (E + e^{-2i\varepsilon})^{-1} \left[\begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix} \otimes \delta_j \right], \tag{20}$$

where $\alpha_j, \beta_j \in \mathcal{A},\ j = 1,2$

In section 3 we derive integral representation for v^ε in terms of Fourier transform of α_j, β_j, which we call spectral function in the sequel. Then we substitute these representations in boundary conditions and deduce an integral equation for the spectral function to satisfy. Further, using coercivity of the form associated with $E + e^{-2i\varepsilon}$, we establish unique solvability of this equation in class $\hat{\mathcal{A}}$, the class of Fourier transforms of functions of $\mathcal{A}$. We conclude this section by calculation of the far-field asymptotics of diffracted field in terms of spectral function.

In section 4 using the same approach we establish existence of Green's tensor for the elastic wedge, which plays a crucial role in section 5 for deriving the radiation conditions. We finish the paper by obtaining the L^2 uniqueness result.

3 Plane wave diffraction

In this section we briefly introduce the spectral function techniques in application to the problem of plane longitudinal (or transversal equally) wave diffraction by an elastic wedge (the more detailed treatment of the techniques is given in [CL99]). The most significant, but not crucial difference between the following and the treatment in [CL99] is not that we consider different problems, but that we reject the condition $\varphi < \pi$. We show, that if hypothesis that neither incident, nor any of multiply reflected waves (which may arise in the case when $\varphi < \pi$) is an incoming grazing one, is satisfied, than there exists an outgoing solution to the problem (S^*).

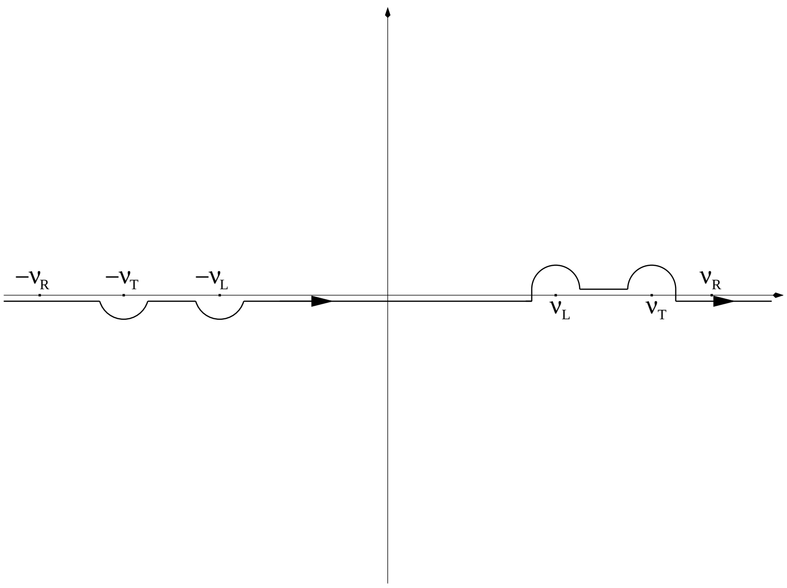

Figure 2: *Contour* Γ_0 *in complex* ξ*-plane*

3.1 Integral system for spectral function

We may rewrite (13) in terms of Fourier transforms of the right-hand side : we have for $\varepsilon \in [0, \pi)$

$$v_j^\varepsilon(x_j, y_j) = \frac{i}{4\pi} e^{2i\varepsilon} \int\limits_{\Gamma_0} e^{ix_j\xi} \sum_{*=L,T} e^{i|y_j|\zeta_*^\varepsilon(\xi)} M_*^\varepsilon(\xi, \mathrm{sign} y_j) \Sigma_j(\xi)\, d\xi, \tag{21}$$

where Γ_0 is the contour pictured on Figure 2, $\zeta_*^\varepsilon(\xi)$ is the branch of $\sqrt{e^{-2i\varepsilon}\nu_*^2 - \xi^2}$ continuous on the contour Γ_0 and possessing non-negative imaginary part on the real axis. Matrices M_*^ε are given by

$$M_L^\varepsilon(\xi, t) = \begin{pmatrix} \dfrac{\xi^2}{\zeta_L^\varepsilon(\xi)} & t\xi \\ t\xi & \zeta_L^\varepsilon(\xi) \end{pmatrix} \tag{22}$$

$$M_T^\varepsilon(\xi, t) = \begin{pmatrix} \zeta_T^\varepsilon(\xi) & -t\xi \\ -t\xi & \dfrac{\xi^2}{\zeta_T^\varepsilon(\xi)} \end{pmatrix} \tag{23}$$

and Σ_j is the Fourier transform of the (α_j, β_j) :

$$\Sigma_j(\xi) = \int\limits_0^\infty e^{-ix_j\xi} \begin{pmatrix} \alpha_j(x_j) \\ \beta_j(x_j) \end{pmatrix} dx_j. \tag{24}$$

The pair (Σ_1, Σ_2) is called spectral function in sequel. Substitution of $v^\varepsilon = v_1^\varepsilon + v_2^\varepsilon$ with v_j represented by (21) into the normal stress boundary operator

$$Bu = (\lambda \mathrm{div} u + 2\mu\varepsilon(u))n, \tag{25}$$

where n is the inward normal to the faces, results in the following

Lemma 3.1. *For $\xi \in \mathbb{C}, \operatorname{Im} \xi < 0$ the Fourier transform of the boundary operator $B(v^\varepsilon)$ on the face Γ_1 is given by*

$$\int_0^\infty e^{-ix\xi} B(v^\varepsilon) dx = \frac{1}{2}[DM^\varepsilon(\Sigma_1) + TM^\varepsilon(\Sigma_2)](\xi), \tag{26}$$

where DM^ε and TM^ε are the integral operators defined for $g \in \hat{\mathcal{A}}^2$ by

$$DM^\varepsilon(g)(\xi) = \int_{\Gamma_0} DM^\varepsilon(\xi,\zeta) g(\zeta)\, d\zeta, \tag{27}$$

$$TM^\varepsilon(g)(\xi) = \int_{\Gamma_0} TM^\varepsilon(\xi,\zeta) g(\zeta)\, d\zeta. \tag{28}$$

The kernels are given by

$$DM^\varepsilon(\xi,\zeta) = \frac{1}{2\pi i}\frac{1}{\xi - \zeta} dm(\zeta e^{i\varepsilon}), \tag{29}$$

with

$$dm(z) = \begin{pmatrix} -1 & A \\ B & -1 \end{pmatrix}, \tag{30}$$

where

$$A(z) = \frac{z}{\zeta_T(z)}(1 - 2\mu Q(z)), \tag{31}$$

$$B(z) = -\frac{z}{\zeta_L(z)}(1 - 2\mu Q(z)), \tag{32}$$

$$Q(z) = \zeta_L(z)\zeta_T(z) + z^2, \tag{33}$$

and

$$TM^\varepsilon(\xi,\zeta) = \frac{1}{2\pi i} \sum_{*=L,T} D^\varepsilon_*(\xi,\zeta) tm_*(\zeta e^{i\varepsilon}, \operatorname{sign}\sin\varphi) \tag{34}$$

with

$$D^\varepsilon_*(\xi,\zeta) = \frac{1}{\xi - (\zeta\cos\varphi + |\sin\varphi|\zeta^\varepsilon_*(\zeta))} \tag{35}$$

and the rank 1 matrices tm_L and tm_T are given by

$$\begin{cases} tm_L(z,t) = (-\frac{\cos\chi}{\sin\chi}\overline{f}_L \ ; \ t\overline{f}_L) \ ; \ \overline{f}_L = \begin{bmatrix} -\mu\sin\psi \\ \mu\cos\psi + \mu - 1 \end{bmatrix} \\ z = \nu_L\cos\chi, \ \zeta_L = -\nu_L\sin\chi, \ \psi = 2(\varphi + t\chi) \end{cases} \tag{36}$$

$$\begin{cases} tm_T(z,t) = (-t\overline{f}_T \ ; -\frac{\cos\chi}{\sin\chi}\overline{f}_T \ ) \ ; \ \overline{f}_T = \begin{bmatrix} -\mu\cos\psi \\ \mu\sin\psi + \mu - 1 \end{bmatrix} \\ z = \nu_T\cos\chi, \ \zeta_T = -\nu_T\sin\chi, \ \psi = 2(\varphi + t\chi) \end{cases} \tag{37}$$

The proof of the lemma is quite technical and absolutely straightforward.

Now we are ready to formulate the problem for the spectral function to satisfy, the following result holds:

Lemma 3.2. *For $\varepsilon \in [0,\pi)$ the function v^ε is the solution to the problem (S^*_ε) if and only if the spectral function $\Sigma = (\Sigma_1, \Sigma_2)$ is the solution of the system*

$$\begin{cases} DM^\varepsilon(\Sigma_1) + TM^\varepsilon(\Sigma_2) = \dfrac{W_1^*}{\xi - \nu_* \cos\theta_{inc}} \\ TM^\varepsilon(\Sigma_1) + DM^\varepsilon(\Sigma_2) = \dfrac{W_2^*}{\xi - \nu_* \cos(\varphi - \theta_{inc})} \end{cases} \tag{38}$$

where

$$W_1^L = -2\begin{pmatrix} \mu \sin 2\theta_{inc} \\ 1 - 2\mu\cos^2\theta_{inc} \end{pmatrix}, \tag{39}$$

$$W_2^L = -2\begin{pmatrix} \mu \sin 2(\varphi - \theta_{inc}) \\ 1 - 2\mu\cos^2(\varphi - \theta_{inc}), \end{pmatrix} \tag{40}$$

$$W_1^T = -2\nu_T \begin{pmatrix} \mu \cos 2\theta_{inc} \\ \mu \sin 2\theta_{inc} \end{pmatrix}, \tag{41}$$

$$W_2^T = 2\nu_T \begin{pmatrix} \mu \cos 2(\varphi - \theta_{inc}) \\ \mu \sin 2(\varphi - \theta_{inc}) \end{pmatrix} \tag{42}$$

Proof. Indeed, the right-hand side in (S^*_ε) is given by:

$$-Bv_L^{inc}|_{\Gamma_1} = -i\begin{pmatrix} \mu \sin 2\theta_{inc} \\ 1 - 2\mu\cos^2\theta_{inc} \end{pmatrix} e^{ix_1 \cos\theta_{inc}}, \tag{43}$$

$$-Bv_L^{inc}|_{\Gamma_2} = -i\begin{pmatrix} \mu \sin 2(\varphi - \theta_{inc}) \\ 1 - 2\mu\cos^2(\varphi - \theta_{inc}) \end{pmatrix} e^{ix_2 \cos(\varphi - \theta_{inc})}, \tag{44}$$

for the case of longitudinal wave incidence, i.e. $* = L$. Taking the Fourier transform of (43),(44) we deduce from (26) identities (38)-(40). Calculations for the transversal wave follow the same scheme. ☐

3.2 Properties of integral and translation operators

Following the strain of thought presented in [CL99] the next results may be obtained:

Lemma 3.3. Properties of the matrix dm

(i) *dm is analytical in $V = \mathbb{C} \setminus ((-\infty,-1) \cup (1,\infty))$ and bounded on each line $z = \xi e^{i\theta}$, with $\xi \in \mathbb{R}$ and $\theta \in (0,\pi)$,*

(ii) *dm possesses only 2 poles on the real axis $\pm\nu_R$, being the roots of the Rayleigh equation* $\det dm(\xi) = 0$,

(iii) *$dm^{-1}(\xi)$ is bounded on V outside a neighbourhood of $\pm\nu_R$.*

Definition 3.4. *H^+ is a Hilbert space of L^2 functions on the positive axis. It consists of all analytical in $\mathbb{C}_- = \{z : \mathrm{Im}\ z < 0\}$ functions h, such that*

$$\sup_{c>0} \int_{\mathbb{R}} |h(x - ic)|^2 dx < \infty. \tag{45}$$

Lemma 3.5. *For all $\theta \in (0, \pi)$ operator DM^θ is bounded as, an operator in H^+:*

$$DM^\theta : (H^+)^2 \to (H^+)^2. \tag{46}$$

Lemma 3.6. *For all $\theta \in (0, \pi)$ operator TM^θ is bounded in H^+. Moreover there exists $c_0(\theta), c_1(\theta), c_2(\theta) > 0$ such that for all $f \in H^+$ its image $TM^\theta f$ possesses an analytical continuation into the domain $W = \{\text{Im } \xi < c_0 + c_1|\text{ Re } \xi|\}$ and for all $\alpha \in (-c_2, c_2)$ $(TM^\theta f)(e^{i\alpha}\xi) \in H^+$.*

Following [CL99] we define translation operators

$$T_*(\xi) = \xi \cos\varphi + \zeta_*(\xi) \sin\varphi \tag{47}$$

and the domains

$$\Omega_* = \{\nu_* \cos z : \ 0 \leq \text{Re } \ z < \varphi\}. \tag{48}$$

In construction of the solution to the plane wave diffraction problem we will also need the sets

$$Z^j(\xi_1, \xi_2) = \bigcup_{l \geq 0} Z_l^j(\xi_1, \xi_2), \ \ j = 1, 2 \tag{49}$$

constructed for arbitrary $\xi_1, \xi_2 \in \mathbb{C}$ following the recursive definition: subsets Z_l^j– the "generation l"– are given by

$$\begin{cases} Z_{l+1}^1 = T_L(Z_l^2 \cap \Omega_L) \cup T_T(Z_l^2 \cap \Omega_T) \\ Z_{l+1}^2 = T_L(Z_l^1 \cap \Omega_L) \cup T_T(Z_l^1 \cap \Omega_T) \end{cases} \tag{50}$$

and initial sets are

$$Z_0^1 = \xi_1, \quad Z_0^2 = \xi_2 \tag{51}$$

The sets we are interested in are generated by $\xi_1 = \cos\theta_{inc}$ and $\xi_2 = \cos(\varphi - \theta_{inc})$. They are the sets of poles of spectral function, corresponding to (multiply) reflected body waves. It may be shown, that these sets are finite.

The generic hypothesis we inherit from [CL99] under which we will prove existence of the solution to the plane wave diffraction problem is

$$\{\nu_L, \nu_T\} \cap Z^j(\cos\theta_{inc}, \cos(\varphi - \theta_{inc})) = \emptyset \ \ \ j = 1, 2 \tag{H}$$

it means, that neither incident, nor any of the reflected waves is an incoming grazing one.

3.3 Isomorphism Theorem

Let us denote the operator corresponding to the entire problem

$$\mathcal{L}^\theta = \begin{pmatrix} DM^\theta & TM^\theta \\ TM^\theta & DM^\theta \end{pmatrix} \tag{52}$$

The following result is the central one in the spectral function techniques and is obtained following the procedure rigorously presented in [CL99]

Theorem 3.7. *For all $\theta \in (0, \pi)$ operator $\mathcal{L}^\theta$ is an isomorphism of the space $(H^+)^2 \oplus (H^+)^2$ into itself.*

Proof. Let us check injectivity first. Suppose $X_1(\xi) = (\hat{\alpha}_1(\xi), \hat{\beta}_1(\xi))$ and $X_2 = (\hat{\alpha}_2(\xi), \hat{\beta}_2(\xi))$ are solutions to

$$\mathcal{L}^\theta \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = 0. \tag{53}$$

We define

$$\alpha = \alpha_1 \delta_1 + \alpha_2 \delta_2 \tag{54}$$

$$\beta = \beta_1 \delta_1 + \beta_2 \delta_2 \tag{55}$$

with α_j, β_j, the inverse Fourier transforms of $\hat{\alpha}_j, \hat{\beta}_j$ and the integration measures δ_j defined by (14). Next we consider a tempered distribution $u \in S'(\mathbb{R}^2)$– the solution of

$$(E + e^{-2i\theta})u = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \tag{56}$$

According to lemma 3.2 we see that (53) implies that $v = u|_\Omega$ is the solution of the homogeneous problem in the wedge

$$\begin{cases} (E + e^{-2i\theta})v = 0 \text{ in } \Omega \\ Bv = 0 \text{ on } \Gamma \end{cases} \tag{57}$$

Regularity results obtained in [Leb95] (Sect. 2.3., Prop. 3) provide us with possibility to apply the theorem of uniqueness for the Neumann problem (57), proved in [CL99], Lemma 4.4. Thus we conclude, that $v = 0$. Next, due to continuity of u and uniqueness for the Dirichlet problem we have $u|_{\mathbb{R}^2 \setminus \Omega} = 0$, and therefore the normal traction jumps over Γ are zeros as well, so $\alpha_j = \beta_j = 0$.

Let us address now surjectivity of $\mathcal{L}^\theta$. For any $Y = (Y_1, Y_2) \in (H^+)^2 \oplus (H^+)^2$ we have representations

$$Y_j(\xi) = \frac{1}{2} \begin{pmatrix} \hat{\alpha}_j(\xi) \\ \hat{\beta}_j(\xi) \end{pmatrix}, \quad j = 1, 2 \tag{58}$$

It is enough to demonstrate, that there exists a simple layer potential, which has normal traction traces on Γ_j from inside of the wedge equal to $(\alpha_j, \beta_j)^T$. To do that, we in some sense perform the above argument in the reverse order: once more consider the Neumann problem inside the wedge

$$\begin{cases} (E + e^{-2i\theta})v_+ = 0 \text{ in } \Omega \\ Bv_+ = \alpha_j \text{ on } \Gamma_j, \quad j = 1, 2. \end{cases} \tag{59}$$

It has a solution according to [CL99] (Lemma 4.4). Next we consider the Dirichlet problem outside the wedge

$$\begin{cases} (E + e^{-2i\theta})v_- = 0 \text{ in } \mathbb{R}^2 \setminus \Omega \\ v_- = v_+ \text{ on } \Gamma, \end{cases} \tag{60}$$

and construct a continuous function in entire space $\mathbb{R}^2$

$$u(x) = \begin{cases} v_+(x), \text{ if } x \in \Omega \\ v_-(x), \text{ if } x \in \mathbb{R}^2 \setminus \Omega \end{cases} \tag{61}$$

To finish the proof it is enough to notice that the above function is a simple layer potential with an L^2 density. $\square$

This theorem has a very important corollary

Theorem 3.8. *Suppose* $Y(\xi) = (Y_1(\xi), Y_2(\xi)) \in \hat{\mathcal{A}}^2 \oplus \hat{\mathcal{A}}^2$ *is an analytic function in* $\mathbb{C}_-^\delta = \{z : \arg z \in (-\pi, \delta)$ *with some* $\delta > 0$*. Suppose as well, that for all* $\varepsilon \in (0, \delta)$ *we have* $Y^\varepsilon = Y(e^{i\varepsilon}\cdot) \in H^+$*, then there exists an* $\hat{\mathcal{A}}^2 \oplus \hat{\mathcal{A}}^2$ *solution of the system*

$$\mathcal{L}^0 X = Y. \tag{62}$$

Proof. First we note that according to the isomorphism theorem for all $\varepsilon \in (0, \delta)$ there exists a solution to the system

$$\mathcal{L}^\varepsilon X^\varepsilon = Y^\varepsilon. \tag{63}$$

Further, it is enough to demonstrate, that $X(\xi) = X^\varepsilon(e^{-i\varepsilon}\xi)$ is indeed the proper solution of (62). Simple change of variables in the definition of the operators DM and TM shows us that the definition of X is correct all ε. The stated in the theorem growth properties of X can be deduced from the properties of the integral operators TM and DM. (see details in [CL99]) □

3.4 Theorem of existence

Recall, that the integral system we are to solve is of a kind

$$\begin{cases} DM\,\Sigma_1(\xi) + TM\,\Sigma_2(\xi) = \frac{W_1}{\xi - z_1} \\ TM\,\Sigma_1(\xi) + DM\,\Sigma_2(\xi) = \frac{W_2}{\xi - z_2} \end{cases} \tag{64}$$

with $z_1 = \nu_* \cos\theta_{inc}, z_2 = \nu_* \cos(\varphi - \theta_{inc})$. We will need the following

Definition 3.9. $\mathcal{H}$ *is a space of analytical in* $\mathbb{U} = \mathbb{C} \setminus (-\infty, 1]$ *functions* h*, such that for all* $\theta \in [0, \pi)$ $h(e^{i\theta}\cdot) \in H^+$*. (recall definition 3.4 of the Hilbert space* H^+*)*

First we introduce the spectral function decomposition. The following lemma can be obtained following the arguments of [CL99]

Lemma 3.10. *There exist two functions* y_1, y_2 *being the finite sum of simple poles*

$$y_j(\xi) = \sum_k \frac{a_j^k}{\xi - z_j^k}, \;\; a_j^k \in \mathbb{C}, \;\; z_j^k \in \mathcal{P}^j \tag{65}$$

(where $\mathcal{P}^j = Z^j(z_1, z_2)$*) such that*

$$\begin{cases} \frac{W_1}{\xi - z_1} - DM\,y_1(\xi) - TM\,y_2(\xi) = u_1(\xi) \in \mathcal{H} \\ \frac{W_2}{\xi - z_2} - TM\,y_1(\xi) - DM\,y_2(\xi) = u_2(\xi) \in \mathcal{H} \end{cases} \tag{66}$$

This lemma and the isomorphism theorem allows us to deduce the following result on existence

Theorem 3.11. *Under the hypothesis (H) there exists an outgoing solution to the problem* (S^*)*. Corresponding to the solution spectral function is of the form:*

$$\Sigma_j = y_j + X_j, \tag{67}$$

with $X_j \in \mathcal{H}$*, such that its boundary values* $X_j(\xi - i0)$*,* $\xi \in \mathbb{R}$ *are analytical for* $\xi \notin \{-\nu_R\} \cup C^j$*, where* $-\nu_R$ *is the simple pole, corresponding to the Rayleigh wave, and* $C^j \subset [-\nu_T, -\nu_L]$ *is the set of ramification point, corresponding to the head wave and its reflections, given by*

$$C^j = Z^j(-\nu_L, -\nu_L) \cup -\nu_T. \tag{68}$$

Any $\xi_0 \in C^j$ *is a square-root type branching point of the spectral function.*

3.5 Far field asymptotics of diffracted field

To calculate asymptotics of $v = v_1 + v_2$, where v_j are defined by (21) with $\varepsilon = 0$, we change variable of integration $\xi = \nu_* \cos\beta$, $d\xi = -\nu_* \sin\beta \, d\beta = \zeta_*(\xi) \, d\beta$ to obtain representation

$$v_1(r\cos\theta, r\sin\theta) = \frac{i}{4\pi} \int\limits_{C_0} \sum_{*=L,T} e^{i\nu_* r(cos\beta\cos\theta - |\sin\theta|\sin\beta)} \nu_*^2 P_*(\beta, t) \Sigma_1(\nu_* \cos\beta) \, d\beta \tag{69}$$

where $t = \operatorname{sign}\sin\theta$ and (orthogonal for real β) projectors P_L and P_T are given by

$$P_L(\beta, t) = \begin{pmatrix} \cos^2\beta & -t\cos\beta\sin\beta \\ -t\cos\beta\sin\beta & \sin^2\beta \end{pmatrix} \tag{70}$$

$$P_T(\beta, t) = \mathbb{I} - P_L(\beta, t) = \begin{pmatrix} \sin^2\beta & t\cos\beta\sin\beta \\ t\cos\beta\sin\beta & \cos^2\beta \end{pmatrix} \tag{71}$$

Contour C_0 is presented on the Figure 3. For $\theta < \pi$, i.e. $t = 1$ we have

$$v_1(r\cos\theta, r\sin\theta) = \frac{i}{4\pi} \int\limits_{C_0} \sum_{*=L,T} e^{i\nu_* r\cos(\beta+\theta)} \nu_*^2 P_*(\beta) \Sigma_1(\nu_* \cos\beta) \, d\beta, \tag{72}$$

here we assume $P_*(\beta) = P_*(\beta, 1)$. In the case $\theta > \pi$ we have

$$v_1(r\cos\theta, r\sin\theta) = \frac{i}{4\pi} \int\limits_{C_0} \sum_{*=L,T} e^{i\nu_* r\cos(\beta-\theta)} \nu_*^2 P_*(-\beta) \Sigma_1(\nu_* \cos\beta) \, d\beta, \tag{73}$$

we have used here identity $P_*(\beta, -1) = P_*(-\beta)$. Now we note that $\Sigma_1(\nu_* \cos\beta)$ is an even function and change variable $\beta = -\beta'$ to obtain the following representation:

$$v_1(r\cos\theta, r\sin\theta) = \frac{i}{4\pi} \int\limits_{C_1} \sum_{*=L,T} e^{i\nu_* r\cos(\beta'+\theta)} \nu_*^2 P_*(\beta') \Sigma_1(\nu_* \cos\beta') \, d\beta', \tag{74}$$

the contour C_1 is pictured on Figure 4.

To calculate asymptotics of integrals (72),(74) we deform contours C_0 and C_1 into stationary ones– contours γ_θ are pictured for both case on Figures 3 and 4. While deforming we might cross some singularities of the spectral function. According to the theorem 3.11 those can only be simple poles (giving rise to plane waves) and ramification points (corresponding to head waves, generated by interaction of the spherical longitudinal wave with the boundary). Diffracted by the vertex spherical waves are given by the contribution of the stationary point $\beta = \pi - \theta$. So we find

$$v_1^{(d)}(x_1, y_1) = \frac{e^{-\pi i/4}}{2\sqrt{2\pi}} \sum_{*=L,T} \nu_*^2 \frac{e^{-i\nu_* r}}{\sqrt{\nu_* r}} P_*(\pi - \theta) \Sigma_1(-\nu_* \cos\theta) \tag{75}$$

In the same way we obtain contribution of Σ_2 in diffracted field, we have:

$$v_2^{(d)}(x_2, y_2) = \frac{e^{-\pi i/4}}{2\sqrt{2\pi}} \sum_{*=L,T} \nu_*^2 \frac{e^{-i\nu_* r}}{\sqrt{\nu_* r}} P_*(\pi - \theta_2) \Sigma_1(-\nu_* \cos\theta_2), \tag{76}$$

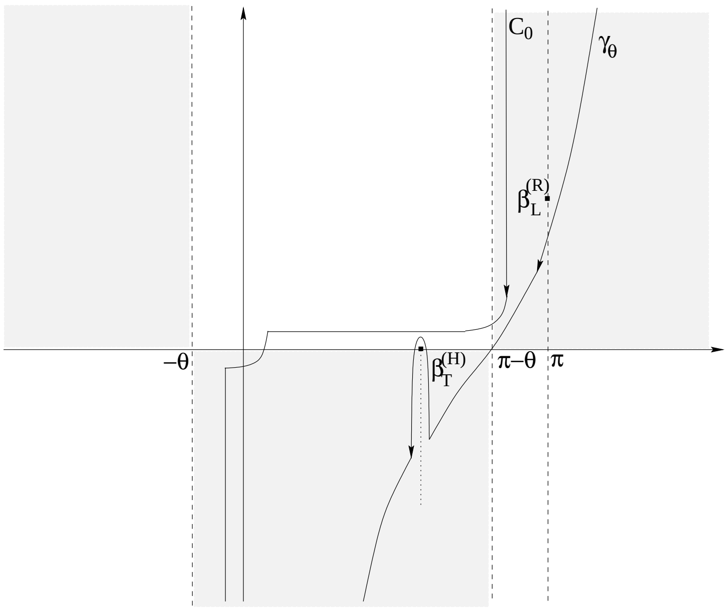

Figure 3: *Contours γ_θ and C_0 in complex β-plane, $\theta < \pi$*

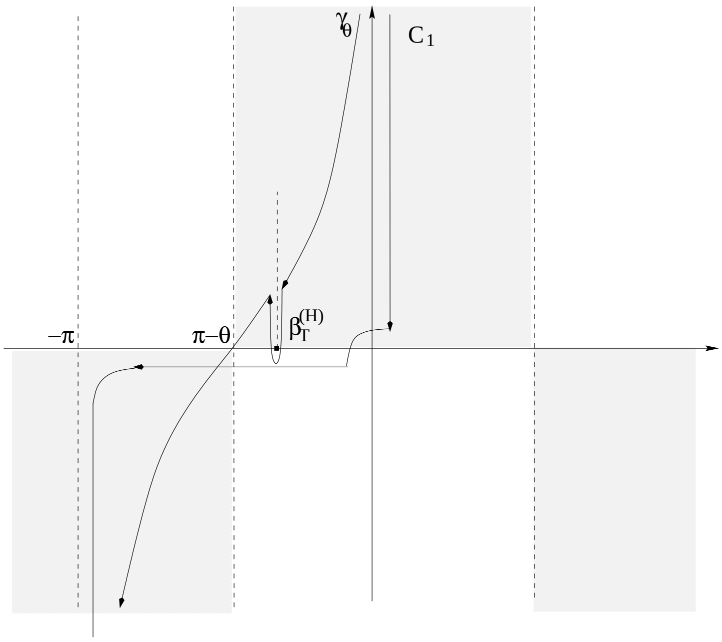

Figure 4: *Contours γ_θ and C_1, $\theta > \pi$*

recall, that $\theta_2 = \varphi - \theta$. The total field, scattered by the tip one obtains by taking the sum of two last expressions.

$$v^{(d)} = v_1^{(d)} + v_2^{(d)}. \tag{77}$$

Now we change variables to polar ones in these expressions to obtain separately main terms of transversal and longitudinal spherical waves diffracted by the tip. We will use the following notation:

$$v^{(d)} = v^{(d),r}\partial_r + v^{(d),\theta}\partial_\theta, \tag{78}$$

where $\partial_r, \partial_\theta$ are canonic coordinate vector fields, and $(v^{(d),r}, v^{(d),\theta})$ are coordinates of displacement in this basis. In this notation we find, that the longitudinal wave scattered by the tip is given by

$$v^{(d),r} = \frac{e^{-\pi i/4}}{2\sqrt{2\pi}}\frac{e^{-ir}}{\sqrt{r}}\big[\Sigma_1^1(-\cos\theta)\cos\theta + \Sigma_1^2(-\cos\theta)\sin\theta$$
$$+\Sigma_2^1(-\cos(\varphi-\theta))\cos(\varphi-\theta) + \Sigma_2^2(-\cos(\varphi-\theta))\sin(\varphi-\theta)\big].$$

and the transversal is

$$v^{(d),\theta} = \frac{e^{-\pi i/4}}{2\sqrt{2\pi}}\nu_T^2\frac{e^{-i\nu_T r}}{\sqrt{\nu_T r^3}}\big[-\Sigma_1^1(-\nu_T\cos\theta)\sin\theta + \Sigma_1^2(-\nu_T\cos\theta)\cos\theta$$
$$+\Sigma_2^1(-\nu_T\cos(\varphi-\theta))\sin(\varphi-\theta) - \Sigma_2^2(-\nu_T\cos(\varphi-\theta))\cos(\varphi-\theta)\big].$$

4 Green's tensor

In this section we deduce the appropriate representation of the free space Green's tensor, calculate it's stresses on the boundary of the wedge. Then we define the notion of Green's tensor in the wedge and calculate the RHS of an integral equation for the corresponding spectral function to satisfy. After the analysis of the RHS regularity we make a conclusion, that Green's tensor for the wedge exists. Finally we calculate asymptotics of the Green's tensor at a distance from the wedge tip.

4.1 Free-space Green's tensor

The time-harmonic elastodynamics free-space Green's tensor $G_f(x, y, x^0, y^0)$ is tensor whose km component represents the k-th component of the displacement caused by a unit point force acting at (x^0, y^0) along the m-axis[1]. Being represented in the matrix form, it satisfies the following equation:

$$(E_{x,y} + 1)G_f(x, y, x^0, y^0) = -\delta_{(x^0,y^0)}\mathbb{I}, \tag{79}$$

where $\mathbb{I}$ is the unit matrix.

Lemma 4.1. *The free space Green's tensor admits the following representation*

$$G_f(x, y, x^0, y^0) = \frac{i}{4\pi}\int_{\Gamma_0} e^{i(x-x^0)\xi}\sum_{*=L,T} e^{i|y-y^0|\zeta_*(\xi)}M_*(\xi, \mathrm{sign}(y-y^0))\,d\xi, \tag{80}$$

where matrices M_* are defined by (22),(23) *with $\varepsilon = 0$.*

[1]this is a traditional definition, not an invariant one

Proof. This lemma can be obtained in the same way, as we've done the computations for spectral function (21). $\square$

One can derive exact formulae for G_f (see [Kup65]), but representation (80) is more convenient for the sequel.

4.2 Existence of Green's tensor for elastic wedge

Definition 4.2. *Green's tensor for an elastic wedge is the sum $G = G_f + G_0$, where G_0 is an outgoing solution to the problem*

$$\begin{cases} (E_{x,y} + 1)G_0 = 0, \\ B_{x,y}G_0|_\Gamma = -B_{x,y}G_f(x, y, x^0, y^0)|_\Gamma \end{cases}. \tag{81}$$

Theorem 4.3. *For arbitrary $(x^0, y^0) \in \Omega$ there exists unique outgoing solution of (81), which can be represented, like in the case of plane wave diffraction, as a sum of two terms, corresponding to faces Γ_j:*

$$G_0 = G_1 + G_2, \tag{82}$$

where

$$G_j(x_j, y_j) = \frac{i}{4\pi} \int_{\Gamma_0} e^{ix_j\xi} \sum_{*=L,T} e^{i|y_j|\zeta_*(\xi)} M_*(\xi, \mathrm{sign} y_j)\Sigma_j(\xi)\, d\xi, \tag{83}$$

Proof. To prove this theorem it's enough to demonstrate, that there exists spectral function $\Sigma = (\Sigma_1, \Sigma_2)$, satisfying an integral equation with the same operator as we've deduced for the plane wave diffraction problem:

$$\begin{cases} DM.\Sigma_1 + TM.\Sigma_2 = 2R_1 \\ TM.\Sigma_1 + DM.\Sigma_2 = 2R_2 \end{cases} \tag{84}$$

here $-R_j$ is the Fourier transform of the stress on the face Γ_j of the free-space Green's tensor:

$$R_j(\xi) = -\int_0^\infty e^{-ix_j\xi}(BG_f)(x_j, 0, x^0, y^0)\, dx_j. \tag{85}$$

So we have to deal with the problem, we know a lot about. In particular to establish it's solvability (or insolvability) it'll be enough to investigate regularity of the right-hand-side, (if necessary and possible) to get rid of it's singularities and finally to apply the isomorphism theorem. So, let us have look what have we got: simple calculations show us that

$$BG_f|_{\Gamma_j} = -\frac{1}{4\pi i} \int_{\Gamma_0} e^{i(x_j - x_j^0)\xi} \sum_{*=L,T} e^{i|y_j^0|\zeta_*(\xi)} N_*(\xi, \mathrm{sign} y_j^0)\, d\xi A^{j-1}. \tag{86}$$

where

$$N_L(\xi, t) = \begin{pmatrix} -2\mu\xi^2 t & 2\mu\xi\zeta_L(\xi) \\ \dfrac{-2\mu\xi^3 + \xi}{\zeta_L(\xi)} & (2\mu\xi^2 - 1)t \end{pmatrix}, \tag{87}$$

$$N_T(\xi, t) = \begin{pmatrix} (2\mu\xi^2 - 1)t & \dfrac{2\mu\xi^3 - \xi}{\zeta_T(\xi)} \\ -2\mu\xi\zeta_T(\xi) & -2\mu\xi^2 t \end{pmatrix} \tag{88}$$

and

$$A = \begin{pmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{pmatrix} \tag{89}$$

is a change of variables (x_1, y_1) to (x_2, y_2) matrix. This non-symmetry in (86) is due to non-invariance of Green's tensor. Evaluating Fourier transform of (86) we finally obtain:

$$R_j(z) = -\frac{1}{4\pi i}\int_{\Gamma_0} \frac{1}{z-\xi} e^{-ix_j^0\xi} \sum_{*=L,T} e^{i|y_j^0|\zeta_*(\xi)} N_*(\xi, \mathrm{sign} y_j^0)\, d\xi A^{j-1}. \tag{90}$$

We can easily see from this formula, that R_j are holomorphic functions in $\mathbb{U} = \mathbb{C}\backslash(-\infty, -1]$ and moreover for all $\varepsilon < \min(\alpha_1, \alpha_2)$, where $\alpha_j = \tan^{-1}|y_j^0/x_j^0| > 0$ we have

$$R_j(ze^{i\varepsilon}) \in H^+. \tag{91}$$

Now we apply the theorem 3.8 and establish existence of solution to (84). □

We should also mention that, the only singularities of $\Sigma_j(z-i0),\ z \in \mathbb{R}$ are the Rayleigh pole $-\nu_R$ and ramification points of C_j.

4.3 Green's tensor far-field asymptotics

We will restrict ourselves to evaluation only of the first term of the asymptotics of $G(x, y, x^0, y^0)$, as $r = \sqrt{x^2+y^2} \to \infty$.

Applying the same change of variable $\xi = \nu_* \cos\beta$ as we have done in calculations of diffraction coefficient in the representation (83) of G_j we will arrive to evaluation of asymptotics of such integrals

$$G_j(x_j, y_j, x^0, y^0) = \frac{i}{4\pi}\sum_{*=L,T}\int_{\gamma_\theta} e^{i\nu_* r\cos(\theta_j+\beta)}\nu_*^2 P_*(\beta)\Sigma_j(\nu_*\cos\beta)\, d\beta, \tag{92}$$

where $x_j = r\cos\theta_j,\ y_j = r\sin\theta_j$ and P_* are defined by (70) with $t = 1$.

So we can say that asymptotics of G_j is the sum of contributions of the stationary point in (92), the singularities of C_j and Rayleigh pole. These terms correspond respectively to the scattered by the tip spherical, (multiply reflected) head and Rayleigh waves. Let us write down all these terms; according to above mentioned structure of the wave, we introduce the following decomposition : $G_j = G_j^{(S)} + G_j^{(H)} + G_j^{(R)}$. We have:

$$\begin{aligned}
G_j^{(S)} &= \frac{e^{-\pi i/4}}{2\sqrt{2\pi}}\sum_{*=L,T}\frac{e^{-i\nu_* r}}{\sqrt{\nu_* r}}\nu_*^2 P_*(\pi-\theta_j)\Sigma_j(-\nu_*\cos\theta_j) + O(r^{-3/2}),\\
G_j^{(R)} &= \sum_{*=L,T} e^{i\nu_* r\cos(\theta_j+\beta_*^{(R)})}\nu_*^2 P_*(\beta_*^{(R)})c_j^{(R)}(x^0, y^0),\\
G_j^{(H)} &= \sum_{\substack{q_n\in C_j\\ *=L,T}}\frac{e^{i\nu_* r\cos(\theta_j+\beta_{n*}^{(H)})}}{r\sqrt{\nu_* r}}P_*(\beta_{n*}^{(H)})d_{j*}^{(H)}(x^0, y^0, q_n)\chi_n(\theta) + O(r^{-5/2}),
\end{aligned}$$

here $\nu_*\cos\beta_*^{(R)} = -\nu_R,\ \nu_*\cos\beta_{n*}^{(H)} = q_n$ and $\chi_n(\theta)$ is a characteristic function of the interval of existence of the head wave :

$$\chi_n(\theta) = H(\pi - \mathrm{Re}\beta_{n*}^{(H)} - \theta)$$

As we see the head wave is negligible in comparison with the spherical waves almost everywhere, i.e. for all admissible angles of observation θ with exception for a finite number of critical directions $\pi - \theta_n$, which correspond to collision of fronts of a head wave and a spherical transversal wave (for $* = L$ the corresponding term in representation of $G^{(H)}$ decays exponentially, due to $C_j \subset [-\nu_T, -\nu_L]$). Moreover in these directions the above presented type of asymptotics becomes inapplicable. To investigate what kind of behaviour G_j performs close to these directions recall that we have the following decomposition of Σ_j:

$$\Sigma_j(\xi) = a(\xi) + b(\xi)\sqrt{(\xi - \theta_{nT})} \tag{93}$$

where a and b are regular functions in small neighbourhood of $\theta_{nT} = \nu_T \cos\theta_n$. Contribution of $a(\nu_T \cos\beta)$ in the integral (92) will give the classical type of asymptotics, while to treat the singular part we turn to uniform stationary phase method (see e.g. [Bor94]), which results in the following type of asymptotics: in assumption $a = 0$ we have uniformly with respect to $|\theta_j - (\pi - \theta_n)|$

$$\begin{aligned} G_j(r,\theta_j) = e^{i\nu_T r(\cos(\theta_j+\theta_n)-1)/2} r^{-\frac{3}{4}} \Big(& D_{1/2}((1+i)\sqrt{\nu_T r(1+\cos(\theta_j+\theta_n))})A \\ & + r^{-1/2} D_{3/2}((1+i)\sqrt{\nu_T r(1+\cos(\theta_j+\theta_n))})B\Big) + O(r^{-3/2}) \end{aligned} \tag{94}$$

where D_α is a parabolic cylinder function of order α. This asymptotics matches of course the above asymptotics of G_j for $|\theta_j - (\pi - \theta_n)| > 0$. In the sequel, when the need of asymptotics of G will arise in most cases we will neglect the head wave, as well, as singular behaviour of diffraction coefficient in critical directions, but of course we will illustrate legitimacy of this simplification of arguments.

Now, let us deal with free space Green's function. The main term of the asymptotics of G_f does not depend on the location of the source (x^0, y^0), so we can suppose $(x^0, y^0) = 0$ in (80). The same change of variable will bring us to the integral

$$G_f(x,y,0,0) = \frac{i}{4\pi} \sum_{*=L,T} \int_{\gamma_\theta} e^{i\nu_* r\cos(\theta+\beta)} \nu_*^2 P_*(\beta)\, d\beta, \tag{95}$$

which we evaluate with the stationary phase method and finally arrive at:

$$G_f(x,y,x^0,y^0) = -\frac{e^{3\pi i/4}}{2\sqrt{2\pi}} \sum_{*=L,T} \frac{e^{-i\nu_* r}}{\sqrt{\nu_* r}} \nu_*^2 P_*(\pi - \theta) + O(r^{-3/2}) \tag{96}$$

5 Theorem of uniqueness in elastic wedge

In this section we will make use of previously constructed Green's tensor, to deduce radiation conditions under which the theorem of uniqueness holds.

Let us recall the Green's formula for elasticity operator: for any bounded domain D and two arbitrary u and v, satisfying regularity properties needed for integration by parts, the following identity holds:

$$-\int_D (Eu, v) = \int_D e(u,v) + \int_{\partial D} (Bu, v), \tag{97}$$

where $e(u,v) = \lambda(\text{div}u)(\text{div}v) + 2\mu\text{tr}(\varepsilon(u)\varepsilon^t(v))$. Changing places of u and v, in this identity and taking difference of a new identity and (97) we arrive at

$$\int_D (Eu,v) - (u,Ev) = -\int_{\partial D} (Bu,v) - (u,Bv) \tag{98}$$

Suppose u is an arbitrary solution of elasticity equation $(E+1)u = 0$ in the wedge, satisfying stress-free boundary conditions on it's faces. Applying (98) to u and to G in domain $\Omega_R = \Omega \cap B_R$ we obtain the following identity :

$$(u^1(x^0,y^0), u^2(x^0,y^0)) = \int_{\partial_R\Omega} (BG,u) - (G,Bu), \tag{99}$$

where $\partial_R\Omega = \Omega \cap S_R$. So, if we knew that the integral in the right-hand side, tends to 0, as $R \to \infty$, we could conclude, that $u(x^0,y^0) = 0$; and if we knew that this is true in some domain, we'd conclude that $u = 0$ globally. Generally speaking, this assumption (that right-hand side vanishes) is the 0-approximation of radiation condition. It's disadvantages are that function G is not available in implicit form and that it supposes fulfillment of too many conditions– for each (x^0,y^0) in some domain... To get rid of these inconveniences we will replace G with its asymptotics in (99), and in order to neglect week terms of asymptotics we will have to suppose u to satisfy suitable boundedness condition as $R \to \infty$. Then, properly grouping different terms which we obtain in calculating asymptotics of BG with the ones we have already derived for G we will deduce radiation conditions and consequently the theorem of uniqueness. Next we demonstrate, that arbitrary L^2 solution to the homogeneous Lame system in the wedge, satisfying stress-free boundary condition, satisfies these radiation conditions as well as growth restrictions.

5.1 Radiation conditions

First, note that normal stress operator B on $\partial_R\Omega$ is given by the formula

$$Bu = -\begin{pmatrix} \cos\theta\partial_x + \mu\sin\theta\partial_y & \mu\sin\theta\partial_x + \lambda\cos\theta\partial_y \\ \lambda\sin\theta\partial_x + \mu\cos\theta\partial_y & \mu\cos\theta\partial_x + \sin\theta\partial_y \end{pmatrix} \tag{100}$$

and it acts on G_j in representation (92) as multiplication of the integrand by a matrix

$$iK_*(\theta,\beta) = -i\nu_* \begin{pmatrix} \cos\theta\cos\beta - \mu\sin\theta\sin\beta & \mu\sin\theta\cos\beta - \lambda\cos\theta\sin\beta \\ -\mu\cos\theta\sin\beta + \lambda\sin\theta\cos\beta & -\sin\theta\sin\beta + \mu\cos\theta\cos\beta \end{pmatrix}. \tag{101}$$

Thus, the first terms of asymptotics of BG_j and G_j will differ only by the matrix $iK_*(\theta,\pi-\theta)$ for spherical wave term and by $iK_*(\theta,\beta_*^{(R)})$ for Rayleigh wave. Multiplier of the same kind $iK_*(\theta,\pi-\theta)$ will differ the first term of asymptotics of BG_f from the one of G_f. Now let us summarize all these considerations in the following formulae:

$$\begin{aligned}
BG_j^{(S)} &= -\frac{e^{3\pi i/4}}{2\sqrt{2\pi}} \sum_{*=L,T} \frac{e^{-i\nu_* r}}{\sqrt{\nu_* r}} i\nu_*^2 K_*^{(S)}(\theta_j) P_*^{(S)}(\theta_j) \Sigma_j(-\nu_*\cos\theta_j) + O(r^{-3/2}) \\
BG_f &= -\frac{e^{3\pi i/4}}{2\sqrt{2\pi}} \sum_{*=L,T} \frac{e^{-i\nu_* r}}{\sqrt{\nu_* r}} i\nu_*^2 K_*^{(S)}(\theta) P_*^{(S)}(\theta) + O(r^{-3/2}) \\
BG_j^{(R)} &= \sum_{*=L,T} e^{-i\nu_* r\cos(\theta_j+\beta_*^{(R)})} i\nu_*^2 K_*^{(R)}(\theta_j) P_*^{(R)} c_j^{(R)}(x^0,y^0)
\end{aligned}$$

we have introduced here the following notation:

$$
\begin{aligned}
K_*^{(S)}(\theta_j) &= K_*(\theta_j, \pi - \theta_j), &(102)\\
K_*^{(R)}(\theta_j) &= K_*(\theta_j, \beta_*^{(R)}), &(103)\\
P_*^{(S)}(\theta_j) &= P_*(\pi - \theta_j), &(104)\\
P_*^{(R)} &= P_*(\beta_*^{(R)}). &(105)
\end{aligned}
$$

We note here that, like in case of G itself, one should replace these asymptotics with uniform ones in vicinities of critical directions which will differ from (94) by the same multiplier as we have for the spherical waves. We omit this step as purely technical.

Before substituting all these asymptotics in (99) let us introduce the following covering of the arch $\partial_R\Omega$:

$$\partial_R\Omega = \partial_R^{(i)}\Omega \cup \partial_R^{(b,1)}\Omega \cup \partial_R^{(b,2)}\Omega \tag{106}$$

where the first is the inner part:

$$\partial_R^{(i)}\Omega = \{(r,\theta) \ : r^{-1+\epsilon} < \theta < \varphi - r^{-1+\epsilon}\} \tag{107}$$

and the later are close to Γ_j parts:

$$
\begin{aligned}
\partial_R^{(b,1)}\Omega &= \{(r,\theta) \ : \theta < 2r^{-1+\epsilon}\}, &(108)\\
\partial_R^{(b,2)}\Omega &= \{(r,\theta) \ : \varphi - \theta < 2r^{-1+\epsilon}\}. &(109)
\end{aligned}
$$

Let us also introduce partition of unity corresponding to (106):

$$\chi^{(i)}(r,\theta) + \chi^{(b,1)}(r,\theta) + \chi^{(b,2)}(r,\theta) = 1, \tag{110}$$

which satisfies $|\partial_\theta \chi^*| \leq C r^{1-\epsilon}$

Now let us substitute all these asymptotic expressions into the right-hand side of (99): we have

$$
\begin{aligned}
\int_{\partial_R\Omega} (BG,u) \ &- \ (G,Bu) = I_S + I_R + &(111)\\
&+ \ O(R^{-1})(\|u\,; L^2(\partial_R^{(i)}\Omega)\| + \|Bu\,; L^2(\partial_R^{(i)}\Omega)\|)\\
&+ \ O(R^{-1/2})(\|u\,; L^1(\partial_R^{(b)}\Omega)\| + \|Bu\,; L^1(\partial_R^{(b)}\Omega)\|)
\end{aligned}
$$

where the main terms of spherical and Rayleigh waves contributions are :

$$
\begin{aligned}
I_S &= \sum_{*=L,T} \int\limits_{\partial_R^{(i)}\Omega} \big((iK_*^{(S)}(\theta) P_*^{(S)}(\theta)\Sigma_*(R,\theta)\,,\, u(R,\theta)) - &(112)\\
&- \ (P_*^{(S)}(\theta)\Sigma_*(R,\theta)\,,\,(Bu)(R,\theta))\big)\chi^{(i)}(R,\theta),\\
I_R &= \sum_{\substack{j=1,2\\ *=L,T}} \int\limits_{\partial_R^{(b,j)}\Omega} \big((iK_*^{(R)}(\theta_j) P_*^{(R)} c_{j,*}(R,\theta_j)\,,\, u(R,\theta_j)) - &(113)\\
&- \ (P_*^{(R)} c_{j,*}(R,\theta_j)\,,\,(Bu)(R,\theta_j))\big)\chi^{(b,j)}(R,\theta),
\end{aligned}
$$

here we have used notation

$$\Sigma_*(R,\theta) = -\frac{e^{3\pi i/4}}{2\sqrt{2\pi}}\frac{e^{-i\nu_* R}}{\sqrt{\nu_* R}}\nu_*^2(\mathbb{I}+A^{j-1}\Sigma_j(-\nu_*\cos\theta)), \tag{114}$$

$$c_{j,*}(R,\theta_j) = \nu_*^2 e^{-i\nu_* R\cos(\theta_j+\beta_*^{(R)})}c_j^{(R)}(x^0,y^0) \tag{115}$$

for spherical and Rayleigh wave components of the Green's function. To deduce this we have used identity $AP_*^{(S)}(\theta_2) = P_*^{(S)}(\theta_1)A$ while changing the variables in expressions for G_2 and BG_2. Note also, that $K_*^{(S)}(\theta)P_*^{(S)}(\theta) = \nu_*^{-1}P_*^{(S)}(\theta)$; therefore moving symmetric matrix $P_*^{(S)}$ to the other argument of pairing we re-write I_S in the following way

$$I_S = \int\limits_{\partial_R^{(i)}\Omega}\sum_{*=L,T}(\Sigma_*(R,\theta)\,,\,i\nu_*^{-1}P_*^{(S)}(\theta)u(R,\theta) - P_*^{(S)}(\theta)(Bu)(R,\theta))\chi^{(i)}(R,\theta) \tag{116}$$

We could now apply Cauchy inequality to estimate I_S by $L^2(\partial_R\Omega)$ norm of the second argument of pairing due to boundedness of the $L^2(\partial_R\Omega)$ norm of the first argument:

$$\|\Sigma_*\,;L^2(\partial_R^{(i)}\Omega)\| \le C, \tag{117}$$

but let us first have a closer look, what we have obtained as the second argument in (116). To do so, we now change variables to polar ones: we will use the following notation:

$$u = u^r\partial_r + u^\theta\partial_\theta, \tag{118}$$

where $\partial_r,\ \partial_\theta$ are canonic coordinate vector fields and (u^r, u^θ) are the displacement coordinates in this basis. In this representation projectors $P_L^{(S)}$ and $P_T^{(S)}$ are given by

$$P_L^{(S)}u = u^r\partial_r,\quad P_T^{(S)}u = u^\theta\partial_\theta, \tag{119}$$

and the normal stress operator B on $\partial_R\Omega$ can be rewritten as follows :

$$Bu|_{\partial_R\Omega} = -\left(\left(\frac{\partial u^r}{\partial r}+\lambda\frac{\partial u^\theta}{\partial\theta}+\lambda\frac{u^r}{r}\right)\partial_r + \mu\left(\frac{\partial u^\theta}{\partial r}+\frac{1}{r^2}\frac{\partial u^r}{\partial\theta}\right)\partial_\theta\right), \tag{120}$$

so, we have

$$iP_L^{(S)}u - P_L^{(S)}Bu = \left(iu^r+\frac{\partial u^r}{\partial r}+\lambda\frac{\partial u^\theta}{\partial\theta}+\lambda\frac{u^r}{r}\right)\partial_r, \tag{121}$$

$$i\nu_T^{-1}P_T^{(S)}u - P_T^{(S)}Bu = \mu\left(i\nu_T u^\theta+\frac{\partial u^\theta}{\partial r}+\frac{1}{r^2}\frac{\partial u^r}{\partial\theta}\right)\partial_\theta. \tag{122}$$

Right-hand sides of these expressions are not exactly of the type we are used to see in formulations of radiation conditions, and indeed it turns out that the third and the fourth terms in (121) as well as the third in (122) are not significant. Doubtlessly the term u^r/r is a weak one in assumption of boundedness of $R^{-1}\|u\,;L^2(\partial_R^{(i)}\Omega)\|$, and the terms with tangent to $\partial_R\Omega$ derivatives can be neglected as well. Let us deal with $\frac{\partial u^\theta}{\partial\theta}$. On this example we will also illustrate possibility to neglect considerations of critical directions. Note, that we can consider contribution of this term on earlier stage, rather then in (116): indeed it's enough to demonstrate, that integral

$$J = \int\limits_{\partial_R^{(i)}\Omega} G^r \frac{\partial u^\theta}{\partial \theta}\chi^{(i)} \tag{123}$$

tends to zero. So, we differentiate by parts to obtain

$$J = -\int\limits_{I^{(i)}} \frac{\partial G^r}{\partial \theta} u^\theta \chi^{(i)}\, R d\theta + O(R^{-1/2-\epsilon/2})\|u\ ; L^2(\partial_R^{(i)}\Omega)\| \tag{124}$$

where the integration interval in variable θ is $I^{(i)} = (R^{-1+\epsilon}, \varphi - R^{-1+\epsilon})$. Let us consider the contribution of vicinity of singular direction $\pi - \theta_n$ in the integral on the right-hand side of (124): differentiating the integral representation of G with respect to θ we obtain the uniform estimate

$$|\frac{\partial G^r}{\partial \theta}| \le CR^{-1/4}|D_{-1/2}((1+i)\sqrt{\nu_T r(1+\cos(\theta_j+\theta_n)))})| + O(R^{-1/2}). \tag{125}$$

We estimate $D_{-1/2}$ with a constant and taking into account that in regular directions G is of order $R^{-1/2}$ we finally estimate

$$|\int\limits_{I^{(i)}} \frac{\partial G^r}{\partial \theta} u^\theta R\, d\theta| \le CR^{-5/4}\|u\ ; L^1(\partial_R^{(i)}\Omega)\| \le CR^{-3/4}\|u\ ; L^2(\partial_R^{(i)}\Omega)\|. \tag{126}$$

So, we conclude

$$|J| \le CR^{-1/2-\varepsilon/2}\|u\ ; L^2(\partial_R^{(i)}\Omega)\|. \tag{127}$$

Now, after getting rid of weak terms we apply Cauchy inequality to (116) and finally evaluate asymptotically

$$\begin{aligned} |I_S| \ &\le C\left(\int_{\partial_R^{(i)}\Omega}\left|iu^r + \frac{\partial u^r}{\partial r}\right|^2 + \int_{\partial_R^{(i)}\Omega}\left|i\nu_T u^\theta + \frac{\partial u^\theta}{\partial r}\right|^2 R^2\right)^{1/2} \\ &+ \quad O(R^{-1/2-\varepsilon/2})\|u;\, L^2(\partial_R^{(i)})\|. \end{aligned} \tag{128}$$

Now, let us consider the contribution (113) of the Rayleigh wave. We are going to treat it in the similar way, just like in case of spherical waves we will extract the strongest term and obtain an estimate analogous to (128). First, we get rid of the cut-off function $\chi^{(b,j)}$ (due to exponential decay of $C_{j,*}$ at distance from the face), next we transform the arch of integration into the interval perpendicular to the face $\delta_R^j\Omega := \{x_j = R,\ 0 \le y_j \le R\sin R^{-1+\varepsilon}\}$. Further, recalling that $c_{j,*}^{(R)}(x^0, y^0)$ is projector onto the eigenvector of $dm(-\nu_R)$ it's obviously enough to consider just the Rayleigh wave w_R, travelling away from the tip, instead of $c_{j,*}$; so let us consider

$$I'_{R,j} := \int\limits_{\delta_R^j\Omega} (Bw_R, u) - (w_R, Bu). \tag{129}$$

Now we note, that

$$Bu|_{\delta_R^j\Omega} = \left(\begin{array}{c} \dfrac{\partial u^x}{\partial x} + \lambda \dfrac{\partial u^y}{\partial y} \\ \mu(\dfrac{\partial u^x}{\partial y} + \dfrac{\partial u^y}{\partial x}) \end{array} \right) \tag{130}$$

and after proper grouping different terms in (129) we find

$$I'_{R,j} = \int_0^{R^\varepsilon} \left((\frac{\partial u^x}{\partial x} w_R^x - u^x \frac{\partial w_R^x}{\partial x}) + \mu((\frac{\partial u^y}{\partial x} w_R^y - u^y \frac{\partial w_R^y}{\partial x})) \right) dy + \alpha_R(w,u) \tag{131}$$

where the second term $\alpha_R(w,u)$ involves normal to the face y-derivatives and is given by

$$\alpha_R(w,u) = \int_0^{R^\varepsilon} \left(\lambda \frac{\partial u^y}{\partial y} w_R^x + \mu w_R^y \frac{\partial u_x}{\partial y} - \lambda u^x \frac{\partial w_R^y}{\partial y} - \mu u^y \frac{\partial w_R^x}{\partial y} \right) dy \tag{132}$$

For the Rayleigh wave we have $\dfrac{\partial w_R}{\partial x} = -i\nu_R w_R$, hence we can estimate the integral in (131) by $CN_R(u)$, where

$$N_R(u) := \|\frac{\partial u}{\partial x} + i\nu_R u;\ L^2(\delta_R^j\Omega)\| \tag{133}$$

And, as we have mentioned earlier, $\alpha_R(w,u)$ can be estimated by the same quantity. Indeed, applying the Green's formula for the curved trapezoid $T_{R_1,R_2} = \{R_1 \le x \le R_2,\ 0 \le y \le x^\varepsilon\}$ we find:

$$|\alpha_{R_1}(w,u) - \alpha_{R_2}(w,u)| \le C\left(N_{R_1}(u) + N_{R_2}(u)\right) + O(e^{-\varepsilon/2R_1}). \tag{134}$$

So, making a natural assumption, that $N_R(u)$ tends to 0, as $R \to \infty$, we conclude that $\alpha_R(w,u)$ has a constant limit.

Now let us consider $\frac{d}{dR}\alpha_R(w,u)$: up to exponentially decaying term we have $\frac{d}{dR}\alpha_R(w,u) \sim \alpha_R(\frac{\partial w}{\partial x}, u) + \alpha_R(w, \frac{\partial u}{\partial x})$. Similar to the above argument shows, that in assumption of $N_R(\frac{\partial u}{\partial x})$ tendency to 0 the derivative $\frac{d}{dR}\alpha_R(w,u)$ tends to a constant as well, and this constant obviously must be 0. So, we have asymptotically

$$\alpha_R(w,u) \sim \alpha_R(w,u) + \frac{1}{2i\nu_R}\frac{d}{dR}\alpha_R(w,u) \sim \frac{1}{2i\nu_R}\alpha_R(w, \frac{\partial u}{\partial x} + i\nu_R u). \tag{135}$$

Thus, recalling representation (132) we conclude, that if $N_R(\frac{\partial u}{\partial x}) \to 0$, as $R \to \infty$, then

$$\alpha_R(w,u) \le C\left(N(\frac{\partial u}{\partial y}) + N(u)\right). \tag{136}$$

So, to establish $\alpha_R(w,u)$ tendency to 0 it will be enough to estimate $N_R(\frac{\partial u}{\partial x})$ and $N(\frac{\partial u}{\partial y})$ with $N(u)$, and this result can be deduced via standard elliptic estimates. Indeed it follows from the

Lemma 5.1. *Suppose u is a solution of the Lame system $(E+1)u = 0$ in the domain $B_{2\rho} := \{|x| \le 2\rho,\ 0 \le y \le 4\rho\}$, satisfying the stress-free boundary condition on $\{y = 0\}$, then the following estimate holds:*

$$\|\nabla u; L^2(i_0)\| \le C \sup_{|r|\le\rho} \|u; L^2(j_r)\|, \tag{137}$$

where $j_r = \{x = r,\ y \le 2\rho\}$, and $i_0 = \{x = 0,\ y \le \rho\}$.

Proof. Let us rescale to new variables $X = \rho^{-1}x$, $Y = \rho^{-1}y$ and introduce $v(X,Y) = u(x,y)$, $B_1 = \rho^{-1}B_\rho$, $I_0 = \rho^{-1}i_0$. We estimate:

$$\|\nabla v; L^2(I_0)\| \le \epsilon\|v;\, H^2(B_1)\| + \frac{C}{\epsilon}\|v;\, L^2(B_2)\| \le$$
$$\le \left((\epsilon C(\rho^2+1) + \frac{C}{\epsilon}\right)\|v;\, L^2(B_2)\| \tag{138}$$

The second inequality is an elliptic estimate, as $(E+\rho^2)v = 0$. Now, changing back to (x,y) variables we find:

$$\rho^{1/2}\|\nabla u; L^2(i_0)\| \le \left((\epsilon(\rho^2+C) + \frac{C}{\epsilon}\right)\rho^{-1}\|u;\, L^2(B_{2\rho})\|$$
$$\le \left((\epsilon(\rho^2+C) + \frac{C}{\epsilon}\right)\rho^{-1/2}\sup_{|r|\le\rho}\|u;\, L^2(j_r)\|, \tag{139}$$

now considering ρ to be big rather than small and choosing $\epsilon = \rho^{-1}$ in the last inequality we obtain the desired estimate. □

Now, combining the derived estimates we obtain the following

Theorem 5.2. *Suppose u satisfies Lame system in the wedge Ω, uniform stress-free boundary condition on Γ, and for some $\epsilon > 0$ it satisfies* **growth conditions at infinity: on some sequence $R \to \infty$**

$$\|u\,; L^2(\partial_R^{(i)}\Omega)\| = o(R^{1/2}), \qquad \|Bu\,; L^2(\partial_R^{(i)}\Omega)\| = o(R), \tag{140}$$
$$\|u\;; L^1(\partial_R^{(b,j)}\Omega)\| = o(R^{1/2}), \qquad \|Bu\;; L^1(\partial_R^{(b,j)}\Omega)\| = o(R^{1/2}), \tag{141}$$

internal radiation conditions

$$\int_{\partial_R^{(i)}\Omega}\left|iu^r + \frac{\partial u^r}{\partial r}\right|^2 \to 0 \qquad \int_{\partial_R^{(i)}\Omega}\left|i\nu_T u^\theta + \frac{\partial u^\theta}{\partial r}\right|^2 R^2 \to 0 \tag{142}$$

and **surface radiation conditions**

$$\int_{\delta_R^{(b)}\Omega}\left|i\nu_R u + \frac{\partial u}{\partial x}\right|^2 \to 0 \tag{143}$$

then $u = 0$.

Remark. We see now that radiation conditions really define the structure of the scattered wave: internal ones (142) mean that inside the wedge the field u comprises scattered by the tip spherical longitudinal and transversal waves, while the boundary conditions (143) allows u to have 2 outgoing Rayleigh waves (one for each face) as it's parts. Radiation conditions in the integral form similar to (142) were first introduced by W. Magnus in [Mag49] for 3-dimensional Helmholtz equation. However an outgoing solution to the problem of plane wave diffraction does not satisfy the radiation conditions (142), but after subtraction of all multiply reflected plane waves it will, although it will not be continuous function anymore.

5.2 L^2 uniqueness

As we have already mentioned the growth conditions (140), (141) are not very restrictive and arbitrary $L^2(\Omega)$ function satisfies them, as well as arbitrary $H^1(\Omega)$ function satisfies radiation conditions (142)-(143). Hence if we could prove that any $L^2(\Omega)$ solution of the homogeneous problem (S) is an $H^1(\Omega)$ function we would demonstrate the absence of L^2 eigenfunctions of the Lame operator with stress-free boundary conditions. So let' prove the following

Lemma 5.3. *Suppose $u \in L^2(\Omega) \cap C^\infty(\Omega)$ is a classical solution of the problem* (4),(5), *then $u \in H^1(\Omega)$*

Proof. It is enough to demonstrate, that $\|u\ ; H^1(\Omega_R)\|$ is bounded, as $R \to \infty$. In order to deal with local estimates let us change variables to slow ones: $x = Ry$, then function $v(y) = u(x)$ satisfies

$$(E + R^2)v = 0 \tag{144}$$

$$Bv|_\Gamma = 0 \tag{145}$$

Let us introduce a cut-off function $\psi \in C^\infty(\mathbb{R}^+)$, which is equal to 1 when it's argument is less than 1, and 0, when it is greater, than 2. We have

$$E(\psi v) = -R^2\psi v + [E, \psi]v, \tag{146}$$

where $[E, \psi] = E\psi - \psi E$ is a first order operator, supported in $\Omega_2 \setminus \Omega_1$. Thus, applying Green's formula (97) to ψv and $\overline{\psi v}$ we obtain:

$$\int_{\Omega_2} e(\psi v, \overline{\psi v}) = \int_{\Omega_2} R^2|\psi v|^2 - \int_{\Omega_2} ([E, \psi]v, \overline{\psi v}) - \int_{\Gamma_{1,2}} ([B, \psi]v, \overline{\psi v}). \tag{147}$$

Here $[B, \psi]$ is a C^∞ matrix, supported in $\Gamma_{1,2} = \Gamma \cap \{1 < r < 2\}$. We estimate the left-hand side of (147) from bellow via Korn inequality (see e.g. [DL72]):

$$\int_{\Omega_2} e(\psi v, \overline{\psi v}) \geq C_K\|\psi v\ ; H^1(\Omega_2)\|^2 - \|\psi v\ ; L^2(\Omega_2)\|^2. \tag{148}$$

The second term on the right-hand side of (147) we integrate by parts and apply Cauchy inequality with small parameter, so we obtain

$$\Big|\int_{\Omega_2} ([E, \psi]v, \overline{\psi v})\Big| \leq \varepsilon_1\|\psi v\ ; H^1(\Omega_2)\|^2 + \frac{C}{\varepsilon_1}\|v\ ; L^2(\Omega_2)\|^2. \tag{149}$$

And the last term in (147) we estimate first by square of the $L^2(\Gamma_{1,2})$ norm, then via Sobolev compact imbedding estimate and finally recalling, that v satisfies stress-free boundary condition (145) we apply standard elliptic estimate:

$$\begin{aligned}\Big|\int_{\Gamma_{1,2}} ([B, \psi]v, \overline{\psi v})\Big| &\leq \varepsilon_2\|v\ ; H^2(V\Gamma_{1,2})\|^2 + \frac{C}{\varepsilon_2}\|v\ ; L^2(\Omega_2)\|^2 \\ &\leq \varepsilon_2\|Ev\ ; L^2(\Omega_2)\|^2 + \frac{C}{\varepsilon_2}\|v\ ; L^2(\Omega_2)\|^2 \\ &= (\varepsilon_2 R^4 + \frac{C}{\varepsilon_2})\|v\ ; L^2(\Omega_2)\|^2. \end{aligned} \tag{150}$$

Here $V\Gamma_{1,2}$ is a vicinity of $\Gamma_{1,2}$. Now combining (148)-(150) with (147) and choosing $\varepsilon_1 = C_K/2$ and $\varepsilon_2 = R^{-2}$ we finally estimate

$$\|v\,; H^1(\Omega_1)\|^2 \le C(R^2+1)\|v\,; L^2(\Omega_2)\|^2. \tag{151}$$

Changing the variables back to fast ones we obtain

$$\|u\,; H^1(\Omega_R)\|^2 \le C\|u\,; L^2(\Omega_{2R})\|^2, \tag{152}$$

which implies the lemma. □

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