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Fredholm determinant representations for correlators of the XY model*

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Abstract

We present results of calculation of the two-point time-dependent temperature correlators of "odd" spin operators in the one-dimensional anisotropic XY spin chain in the external field. In the thermodynamic limit the correlators are represented in terms of Fredholm determinants of linear integral operators with matrix kernels.

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I. INTRODUCTION

The one-dimensional XY spin chain [1, 2] was one of the first integrable models whose correlation functions had been studied by exact treatment [3, 4]. In the quantum spin 1/2 chains the most interesting and complicated correlators are two-point time-dependent temperature ones of the "odd" spin operators. Though some expressions were found for these correlators in the XY spin chain a long ago [5], their asymptotic behaviour were possible to study only at the zero temperature case [5, 6].

The standard modern approach to the correlation functions of quantum integrable models is based on their representations, in the thermodynamic limit, as Fredholm determinants of linear integral operators [7]. The integral operators entering the Fredholm determinant representations (FDR) are of the special kind: they are the "integrable" integral operators [8]. With an integrable integral operator a matrix Riemann-Hilbert problem can be associated and an asymptotic expansion of the correlators at large temporal and spacial separations can be constructed. For the temperature correlators in the isotropic XY model this programm was fulfilled in Refs. [9–11].

In this letter we present FDR for time-dependent temperature correlators of the "odd" spin operators in the anisotropic XY model. It is to be mentioned that the models whose correlators have be obtained in this form are solvable by the Bethe Ansatz method. Though the anisotropic XY model admits solution in terms of free fermions, its eigenstates are not Bethe vectors. The method of derivation of FDR for correlators in the XY model, based on the Grassmann coherent states, was suggested in Ref. [12] on the example of equal-time correlators. We have used this method in Ref. [13] to derive time-dependent correlators but the dependence on time of integral operators entering FDR found therein appears to be too complicated for applying the approach based on the Riemann-Hilbert problem directly.

In the present paper for the model on the finite lattice of length M we obtain correlators in terms of determinants of $2M \times 2M$ matrices with the exponential dependence on time. In these formulae the matrices generating the dependence on time and temperature can be recognized as Dirac operators (in the momentum representation) of the Jordan-Wigner lattice fermions. In the thermodynamic limit we obtain FDR for the correlators. The integral operators are integrable, as well as in Ref. [13], but with kernels possessing simple dependence on time, similarly to the isotropic case [9, 10].

In the next Section the solution of the XY model is reviewed and the quantities under investigation are defined. In the Section III the method based on the Grassmann coherent states is explained and necessary matrix elements of operators are given. The Section IV is devoted to derivation of the correlators in the model on the finite lattice. The results for the correlators in the thermodynamic limit are given in the Section V.

II. THE XY MODEL

The Hamiltonian of the anysotropic XY model describing near-neighbor interaction of M spins 1/2 placed in the sites of the one-dimensional periodic lattice is

$$\mathbf{H} = -\sum_{m=1}^{M} \left[\sigma_m^+ \sigma_{m+1}^- + \sigma_m^- \sigma_{m+1}^+ + \gamma \left(\sigma_m^+ \sigma_{m+1}^+ + \sigma_m^- \sigma_{m+1}^- \right) + \frac{h}{2}\, \sigma_m^z \right]. \tag{1}$$

The operators $\sigma_m^\pm = (\sigma_m^x \pm i\sigma_m^y)/2$ and σ_m^z are the Pauli spin operators which correspond to m-th site of the lattice. The constant h is the external magnetic field along z-direction of the total spin and γ is the parameter of anysotropy. The case $\gamma = 0$ corresponds to the isotropic XY (the XX0) model. In what follows we assume no restriction on the values of h and γ. The solution of the model is based on Jordan-Wigner transformation followed by Fourier and Bogolyubov transformations [1, 2]. Below we sketch the solution following Refs. [5, 12].

The Jordan-Wigner transformation

$$c_m = \exp\left(\mathrm{i}\pi \sum_{j=1}^{m-1} n_j \right) \sigma_m^+ , \qquad c_m^\dagger = \sigma_m^- \exp\left(-\mathrm{i}\pi \sum_{j=1}^{m-1} n_j \right), \tag{2}$$

where $n_m = (1 - \sigma_m^z)/2 = \sigma_m^- \sigma_m^+ = c_m^\dagger c_m$ is the local density operator with respect to the ferromagnetic state with all spins up, introduces fermion operators on the lattice with the canonical anticommutation relations: $\{c_m, c_n^\dagger\} = \delta_{mn}$. The periodic boundary conditions for the "odd" spin operators $\sigma_{M+1}^\pm = \sigma_1^\pm$ turn into the conditions $c_{M+1} = (-)^{\mathbf{N}} c_1$ and $c_{M+1}^\dagger = c_1^\dagger (-)^{\mathbf{N}}$ where $(-)^{\mathbf{N}}$ is the parity operator $(-)^{\mathbf{N}} \equiv \exp\{\pm \mathrm{i}\pi \mathbf{N}\} = \sigma_1^z \sigma_2^z \ldots \sigma_M^z$. The fermion number operator $\mathbf{N}$ is not an integral of motion for the anisotropic XY model but the operator $(-)^{\mathbf{N}}$ is that one, $[\mathbf{H}, (-)^{\mathbf{N}}] = 0$. Hence,

$$\mathbf{H} = \mathbf{H}^+ \mathbf{P}^+ + \mathbf{H}^- \mathbf{P}^-, \qquad \mathbf{P}^\pm = \frac{1}{2}\Big[1 \pm (-)^{\mathbf{N}} \Big]. \tag{3}$$

where the both Hamiltonians $\mathbf{H}^\pm$ may be written formally in the same form but they differ from each other by the boundary conditions for the fermions:

$$\mathbf{H}^{\pm} = -\sum_{m=1}^{M}\Big[c^{\dagger}_{m}c_{m+1} + c^{\dagger}_{m+1}c_{m} + \gamma\Big(c_{m+1}c_{m} + c^{\dagger}_{m}c^{\dagger}_{m+1}\Big) - h\Big(c^{\dagger}_{m}c_{m} - \frac{1}{2}\Big)\Big],$$

$$c_{M+1} = \mp c_1\,, \qquad c^{\dagger}_{M+1} = \mp c^{\dagger}_1\,. \tag{4}$$

Therefore, the Fourier transformation to the momentum representation is different for these Hamiltonians.

Let us introduce two sets $\mathbb{X}^{\pm} = \{p_1, p_2, \ldots, p_M\}$ of values of the momentum p where the numbers p_a are equal to

$$p_a = -\pi - \frac{\pi}{M} + \frac{2\pi a}{M} \quad \text{for } \mathbb{X}^{+}, \qquad p_a = -\pi + \frac{2\pi a}{M} \quad \text{for } \mathbb{X}^{-}. \tag{5}$$

These values of the momentum are choosen so that $p \in (-\pi, \pi]$ and they satisfy the equations $\exp(\mathrm{i}pM) = \mp 1$ for $p \in \mathbb{X}^{\pm}$. In terms of canonical fermion operators in the momentum representation

$$c_p = \frac{\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{M}}\sum_{m=1}^{M}\mathrm{e}^{-\mathrm{i}(m-1)p}c_m\,, \qquad c^{\dagger}_p = \frac{\mathrm{e}^{-\mathrm{i}\pi/4}}{\sqrt{M}}\sum_{m=1}^{M}\mathrm{e}^{\mathrm{i}(m-1)p}c^{\dagger}_m\,, \qquad p \in \mathbb{X}^{\pm} \tag{6}$$

the Hamiltonians $\mathbf{H}^{\pm}$ acquire the form

$$\mathbf{H}^{\pm} = \sum_{p\in\mathbb{X}^{\pm}}\Big[\varepsilon(p)\,\Big(c^{\dagger}_p c_p - \frac{1}{2}\Big) + \frac{\varkappa(p)}{2}\Big(c^{\dagger}_p c^{\dagger}_{-p} + c_{-p}c_p\Big)\Big] \tag{7}$$

where

$$\varepsilon(p) = h - 2\cos p\,, \qquad \varkappa(p) = 2\gamma\sin p\,. \tag{8}$$

The Hamiltonians (7) can be diagonalized by means of the Bogolyubov transformation to the new canonical fermion operators

$$C_p = \cos\frac{\theta(p)}{2}\,c_p - \sin\frac{\theta(p)}{2}\,c^{\dagger}_{-p}\,, \qquad C^{\dagger}_{-p} = \sin\frac{\theta(p)}{2}\,c_p + \cos\frac{\theta(p)}{2}\,c^{\dagger}_{-p} \tag{9}$$

what yields

$$\mathbf{H}^{\pm} = \sum_{p\in\mathbb{X}^{\pm}}\Lambda(p)\Big(C^{\dagger}_p C_p - \frac{1}{2}\Big). \tag{10}$$

The angle $\theta(p)$ of the Bogolyubov transformation (9) and the one-particle dispertion $\Lambda(p)$ are defined by the relations:

$$\cos\theta(p) = \frac{\varepsilon(p)}{\Lambda(p)}\,, \qquad \sin\theta(p) = -\frac{\varkappa(p)}{\Lambda(p)}\,, \qquad \Lambda(p) = \begin{cases}\sqrt{\varepsilon^2(p) + \varkappa^2(p)} & p \neq 0, \pi \\ \varepsilon(p) & p = 0, \pi\end{cases}. \tag{11}$$

Note that if one chooses $-\pi < \theta(p) < \pi$ then Eqns. (8) and (11) fix the angle $\theta(p)$ to be an odd 2π-periodic function of the momentum, $\theta(p) = -\theta(-p)$.

The two-point correlators of the "odd" spin operators $\sigma_m^{\pm}$ in the model on the finite lattice are defined as

$$\big\langle\!\big\langle \sigma^a_{m+1}(t)\, \sigma^b_1(0) \big\rangle\!\big\rangle_M = \frac{\mathrm{Tr}\left\{ \mathrm{e}^{-(\beta-\mathrm{i}t)\mathbf{H}} \sigma^a_{m+1} \mathrm{e}^{-\mathrm{i}t\mathbf{H}} \sigma^b_1 \right\}}{\mathrm{Tr}\left\{\mathrm{e}^{-\beta\mathbf{H}}\right\}}, \qquad a, b = \pm, \tag{12}$$

where t is time and β is reciprocal temperature. The trace is taken over the state space of the model. Both the numerator and the denominator (the partition function) in Eqn. (12) may be written, due to (3), as the sum of four contributions and, therefore, one can write

$$\big\langle\!\big\langle \sigma^a_{m+1}(t)\, \sigma^b_1(0) \big\rangle\!\big\rangle_M = \frac{G^{(ab)+}_{\mathrm{F}} + G^{(ab)-}_{\mathrm{F}} + G^{(ab)+}_{\mathrm{B}} - G^{(ab)-}_{\mathrm{B}}}{Z^+_{\mathrm{F}} + Z^-_{\mathrm{F}} + Z^+_{\mathrm{B}} - Z^-_{\mathrm{B}}}. \tag{13}$$

The contributions into the partition function $Z = (Z^+_{\mathrm{F}} + Z^-_{\mathrm{F}} + Z^+_{\mathrm{B}} - Z^-_{\mathrm{B}})/2$ are defined as

$$Z^{\pm}_{\mathrm{F}} = \mathrm{Tr}\left\{ \mathrm{e}^{-\beta \mathbf{H}^{\pm}} \right\}, \qquad Z^{\pm}_{\mathrm{B}} = \mathrm{Tr}\left\{ (-1)^{\mathbf{N}} \mathrm{e}^{-\beta \mathbf{H}^{\pm}} \right\}. \tag{14}$$

Here $Z^{\pm}_{\mathrm{F}}$ are the "fermionic" contributions and $Z^{\pm}_{\mathrm{B}}$ are the "bosonic" ones since they are equal to the partition functions of free fermions and the reciprocial partition functions of free bosons respectively

$$Z^{\pm}_{\mathrm{F}} = \prod_{p \in \mathbb{X}^{\pm}} \left(2\cosh \frac{\beta \Lambda(p)}{2} \right), \qquad Z^{\pm}_{\mathrm{B}} = \prod_{p \in \mathbb{X}^{\pm}} \left(2\sinh \frac{\beta \Lambda(p)}{2} \right). \tag{15}$$

Similarly to Eqn. (14), the "fermionic" and "bosonic" contributions to the numerator in Eqn. (13) are defined as

$$G^{(ab)\pm}_{\mathrm{F}} = \mathrm{Tr}\left\{ \mathrm{e}^{-(\beta-\mathrm{i}t)\mathbf{H}^{\pm}} \sigma^a_{m+1} \mathrm{e}^{-\mathrm{i}t\mathbf{H}^{\mp}} \sigma^b_1 \right\}, \qquad G^{(ab)\pm}_{\mathrm{B}} = \mathrm{Tr}\left\{ (-)^{\mathbf{N}} \mathrm{e}^{-(\beta-\mathrm{i}t)\mathbf{H}^{\pm}} \sigma^a_{m+1} \mathrm{e}^{-\mathrm{i}t\mathbf{H}^{\mp}} \sigma^b_1 \right\}. \tag{16}$$

Below we present the method of exact calculation of the contributions $G^{(ab)\pm}_{\mathrm{F,B}}$. These results allows us to obtain the representations for the correlators (12) in the thermodynamic limit, $M \to \infty$.

III. GRASSMANN COHERENT STATES

Let us introduce Grassmann coherent states [14]

$$\big|\phi_{(\pm)}\big\rangle = \exp\left(\sum_{p \in \mathbb{X}^{\pm}} c^{\dagger}_p \phi_p \right) \big|0\big\rangle, \qquad \big\langle \phi^*_{(\pm)}\big| = \big\langle 0\big| \exp\left(\sum_{p \in \mathbb{X}^{\pm}} \phi^*_p c_p \right), \tag{17}$$

which are the eigenstates of the operators c_p and $c_p^\dagger$ correspondingly

$$c_p \left|\phi_{(\pm)}\right\rangle = \phi_p \left|\phi_{(\pm)}\right\rangle, \qquad \left\langle\phi^*_{(\pm)}\right| c_p^\dagger = \phi_p^* \left\langle\phi^*_{(\pm)}\right|, \qquad p \in \mathbb{X}^\pm. \tag{18}$$

The variables ϕ_p and ϕ_p^* are elements of Grassmann algebra and they anticommute with each other and with all fermionic operators. There is no complex involution on the Grassmann algebra, ϕ_p and ϕ_p^* are independent (real) variables. The star just denotes that the variable is related to the bra-vector.

Using the Grassmann coherent states one can calculate traces of the operators acting in the state space of the model. Trace of an operator $\mathbf{A}$ can be written in the form

$$\mathrm{Tr}\,\mathbf{A} = \int \mathrm{d}\phi\,\mathrm{d}\phi^* \exp\left(\sum_{p\in\mathbb{X}+} \phi_p^*\phi_p\right) \left\langle\phi^*_{(+)}\right| \mathbf{A} \left|\phi_{(+)}\right\rangle, \qquad \mathrm{d}\phi\,\mathrm{d}\phi^* \equiv \prod_{p\in\mathbb{X}+} \mathrm{d}\phi_p \mathrm{d}\phi_p^*. \tag{19}$$

The similar formula can be written down using the Grassmann vaiables corresponding to the set $\mathbb{X}^-$. Evaluation of matrix elements of complicated operators, like in Eqns. (16), can be done using decomposition of the unity operator

$$\mathbf{I} = (-1)^M \int \mathrm{d}\phi\,\mathrm{d}\phi^* \exp\left(-\sum_{p\in\mathbb{X}^\pm} \phi_p^*\phi_p\right) \left|\phi_{(\pm)}\right\rangle\left\langle\phi^*_{(\pm)}\right|. \tag{20}$$

To write scalar products of coherent states and matrix elements of various operators in the basis of the coherent states we shall use shorthand notations for the sums, like those in Eqns. (19) and (20), as scalar products of vectors. Let $\phi = (\phi_{p_1}, \ldots, \phi_{p_M})$ and $\phi^* = (\phi^*_{p_1}, \ldots, \phi^*_{p_M})$ denote M-vectors whose elements are Grassmann variables and all the numbers p_a, given by formulae (5), either from the set $\mathbb{X}^+$ or from the set $\mathbb{X}^-$. In terms of these notations the scalar products of the coherent states read

$$\left\langle\phi^*_{(\pm)}\middle|\phi_{(\pm)}\right\rangle = \exp\bigl(\phi^*\phi\bigr), \qquad \left\langle\phi^*_{(\pm)}\middle|\phi_{(\mp)}\right\rangle = \exp\bigl(\phi^* L^{\pm 1}\phi\bigr). \tag{21}$$

Entries of the matrices L and L^{-1} are determined by the function

$$L(p,q) = \frac{2}{M(1 - \mathrm{e}^{-\mathrm{i}(p-q)})} \tag{22}$$

and they are given as

$$(L)_{ab} = L(p_a, q_b), \qquad (L^{-1})_{ab} = L(q_a, p_b), \qquad p \in \mathbb{X}^+,\ q \in \mathbb{X}^-. \tag{23}$$

The formulae (23) follow from the identity

$$\sum_{q\in\mathbb{X}^{\mp}} L(p,q)L(q,p') = \delta(p,p') \equiv \begin{cases} 1, & p=p' \\ 0, & p\neq p' \end{cases}, \qquad p,p'\in\mathbb{X}^{\pm}. \tag{24}$$

which can be regarded as the unitarity property of the function $L(p,q)$.

The property (24) together with (22) implies that the matrix L is a unitary matrix with respect to complex conjugation. In addition, there is one more unitarity of the matrix L, which is related to the involutive automorphism of the Grassmann algebra with respect to inversion of the sign of the momentum, $\phi_p \to \phi_{-p}$, provided one assumes that $\phi_{-\pi} \equiv \phi_\pi$. Inversion of the sign of the momentum admits the following matrix realization

$$(\Omega^{(\pm)})_{ab} = \delta(-p_a, p'_b), \qquad p,p'\in\mathbb{X}^{\pm}; \qquad \Omega^{(\pm)T} = \Omega^{(\pm)}, \qquad (\Omega^{(\pm)})^2 = I. \tag{25}$$

Due to the property $L(-p,-q) = L(q,p)$ one has

$$\overline{L}^{\,T} = L^{-1}, \qquad \overline{L} \equiv \Omega^{(+)} L\, \Omega^{(-)}. \tag{26}$$

Let us now turn to the matrix elements of the operators entering into the definitions of the correlators. The matrix elements of the spin operators $\sigma^{\pm}_m$ between the coherent states corresponding to different sets of the momenta are given by the formulae [12]:

$$\begin{aligned} \left\langle \phi^*_{(\pm)}\right| \sigma^+_{m+1} \left|\phi_{(\mp)}\right\rangle &= \frac{\mathrm{e}^{-\mathrm{i}\pi/4}}{\sqrt{M}} \left(\sum_{p\in\mathbb{X}^{\mp}} \mathrm{e}^{imp}\phi_p\right) \exp\Big\{\phi^*(P^{(\pm)})^m L^{\pm 1} (P^{(\mp)})^{-m}\phi\Big\}, \\ \left\langle \phi^*_{(\pm)}\right| \sigma^-_{m+1} \left|\phi_{(\mp)}\right\rangle &= \frac{\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{M}} \left(\sum_{p\in\mathbb{X}^{\pm}} \mathrm{e}^{-imp}\phi^*_p\right) \exp\Big\{\phi^*(P^{(\pm)})^m L^{\pm 1} (P^{(\mp)})^{-m}\phi\Big\}, \end{aligned} \tag{27}$$

where $P^{(\pm)}$ are $M\times M$ diagonal matrices with entries

$$(P^{(\pm)})_{ab} = \exp(-\mathrm{i}p_a)\,\delta_{ab}, \qquad p\in\mathbb{X}^{\pm}. \tag{28}$$

The matrix elements of the operators $\exp(-z\mathbf{H}^{\pm})$ between the coherent states corresponding to the same sets of the momenta are given as

$$\left\langle \phi^*_{(\pm)}\right| \mathrm{e}^{-z\mathbf{H}^{\pm}} \left|\phi_{(\pm)}\right\rangle = \det{}_M U^{(\pm)}_{-z} \exp\big(\zeta^* D^{(\pm)}_z \zeta\big), \tag{29}$$

where the variables ζ_p, ζ^*_p and ϕ_p, ϕ^*_p are related as follows

$$\begin{pmatrix} \zeta \\ \zeta^* \end{pmatrix} = \mathcal{O}^{(\pm)} \begin{pmatrix} \phi \\ \phi^* \end{pmatrix}, \qquad \mathcal{O}^{(\pm)} = \frac{1}{\sqrt{2}} \begin{pmatrix} I & \mathrm{i}\Omega^{(\pm)} \\ \mathrm{i}\Omega^{(\pm)} & I \end{pmatrix}. \tag{30}$$

The matrices $U_z^{(\pm)}$ and $D_z^{(\pm)}$ are $M\times M$ diagonal ones with entries

$$(U_z^{(\pm)})_{ab} = u_z(p_a)\,\delta_{ab}, \qquad (D_z^{(\pm)})_{ab} = d_z(p_a)\,\delta_{ab}, \qquad p\in\mathbb{X}^{\pm}. \tag{31}$$

The functions $u_z(p)$ and $d_z(p)$ are defined as

$$u_z(p) = \cosh\frac{z\Lambda(p)}{2} - \frac{\varepsilon(p)-\mathrm{i}\varkappa(p)}{\Lambda(p)}\sinh\frac{z\Lambda(p)}{2}, \qquad d_z(p) = \frac{u_z(p)}{u_{-z}(p)}, \tag{32}$$

where the functions $\varepsilon(p)$ and $\varkappa(p)$ are given by Eqn. (8). The expressions (29) can be derived, e.g., by means of constructing the coherent states for the "new" fermion operators (9), which diagonalize the Hamiltonians, and calculating the scalar products between the "new" and the "old" coherent states [12].

Thus, all matrix elements of the operators entering the correlation functions are known and using formulae (19), (20), (27) and (29) one can, in pricipal, calculate any correlation function of the model.

IV. CORRELATORS OF THE MODEL ON THE FINITE LATTICE

Let us outline the calculation of the contributions $G_{\mathrm{F}}^{(ab)\pm}$ and $G_{\mathrm{B}}^{(ab)\pm}$ (16) on the example of the contribution $G_{\mathrm{F}}^{(ab)+}$ which can be written, due to (19) and (20), as the integral over Grassmann variables

$$\begin{aligned} G_{\mathrm{F}}^{(ab)+} = (-1)^M \int \mathrm{d}\psi\,\mathrm{d}\eta^*\,\mathrm{d}\eta\,\mathrm{d}\xi^*\,\mathrm{d}\xi\,\mathrm{d}\phi^*\,\mathrm{d}\phi\,\mathrm{d}\psi^*\,\exp\bigl(\eta^*\psi - \xi^*\eta - \phi^*\xi - \psi^*\phi\bigr) \\ \times \bigl\langle \eta^*_{(+)}\bigr|\,\mathrm{e}^{-(\beta-\mathrm{i}t)\mathbf{H}^+}\bigl|\eta_{(+)}\bigr\rangle\bigl\langle \xi^*_{(+)}\bigr|\,\sigma^a_{m+1}\,\bigl|\xi_{(-)}\bigr\rangle\bigl\langle\phi^*_{(-)}\bigr|\,\mathrm{e}^{-\mathrm{i}t\mathbf{H}^-}\,\bigl|\phi_{(-)}\bigr\rangle\bigl\langle\psi^*_{(-)}\bigr|\,\sigma_1^b\,\bigl|\psi_{(+)}\bigr\rangle \end{aligned} \tag{33}$$

$(a,b=\pm)$. After substitution of the expressions (27) and (29) for the matrix elements in Eqn. (33) and simple change of the Grassmann variables the contribution under the question takes the form

$$\begin{aligned} G_{\mathrm{F}}^{(ab)+} = \det{}_M U^{(+)}_{-\beta+\mathrm{i}t}\,\det{}_M U^{(-)}_{-\mathrm{i}t}\,\frac{\partial}{\partial\alpha}\int \mathrm{d}\psi\,\mathrm{d}\eta^*\,\mathrm{d}\eta\,\mathrm{d}\xi^*\,\mathrm{d}\xi\,\mathrm{d}\phi^*\,\mathrm{d}\phi\,\mathrm{d}\psi^*\,\exp\Bigl(-\eta^*\psi - \xi^*\eta \\ - \phi^*\xi - \psi^*\phi + \omega^* D^{(+)}_{\beta-it}\,\omega + \zeta^* D^{(-)}_{it}\,\zeta + \xi^*\overline{H}\,\xi + \psi\,F\,\psi^* + \alpha f^{(ab)}\Bigr)\Bigr|_{\alpha=0} \end{aligned} \tag{34}$$

where the variables ω, ω^* and ζ, ζ^* are expressed through η, η^* and ϕ, ϕ^* correspondingly by Eqn. (30). The quatities $f^{(ab)}$ are defined as follows

$$f^{(-+)} = -\xi^* R^{(+)}\psi\,, \quad f^{(+-)} = \xi\,\overline{R}^{(-)}\psi^*\,, \quad f^{(++)} = \mathrm{i}\xi\,\Omega^{(-)}K^T\psi\,, \quad f^{(--)} = \mathrm{i}\xi^* K\Omega^{(-)}\psi^* \tag{35}$$

where the following matrices are introduced

$$(R^{(\pm)})_{ab} = \frac{\mathrm{e}^{-\mathrm{i}\frac{m}{2}(p_a+p'_b)}}{M}\,, \quad p, p' \in \mathbb{X}^{\pm}; \qquad (K)_{ab} = \frac{\mathrm{e}^{-\mathrm{i}\frac{m}{2}(p_a+q_b)}}{M}\,, \quad p \in \mathbb{X}^{+},\ q \in \mathbb{X}^{-}. \tag{36}$$

The matrices F and H in (34) are "dressed" forms of the matrix L (22)

$$F = (P^{(+)})^{\frac{m}{2}}\,\overline{L}\,(P^{(-)})^{-\frac{m}{2}}, \qquad H = (P^{(+)})^{-\frac{m}{2}}\,\overline{L}\,(P^{(-)})^{\frac{m}{2}} \tag{37}$$

where the diagonal matrices $P^{(\pm)}$ are given by (28). As in Eqn. (26), the bar denotes inversion of signs of all momenta of a function determining entries of a matrix.

The Grassmann integral (34) is the Gaussian one and it can be calculated exactly [15]. Evaluating the integrals over the variables η, η^*, ϕ and ϕ^* and combining the rest variables in $4M$-vector $\Psi = (\psi, \xi^*, \xi, \psi^*)$ one obtaines that

$$\begin{aligned} G_{\mathrm{F}}^{(ab)+} &= \det\nolimits_M U^{(+)}_{\beta-\mathrm{i}t}\,\det\nolimits_M U^{(-)}_{\mathrm{i}t}\,\frac{\partial}{\partial\alpha}\int d\Psi\,\exp\left(\frac{1}{2}\Psi\mathsf{A}_{\mathrm{F}}^{(ab)+}(\alpha)\Psi\right)\bigg|_{\alpha=0} \\ &= \det\nolimits_M U^{(+)}_{\beta-\mathrm{i}t}\,\det\nolimits_M U^{(-)}_{\mathrm{i}t}\,\frac{\partial}{\partial\alpha}\,\det\nolimits^{1/2}_{4M}\,\mathsf{A}_{\mathrm{F}}^{(ab)+}(\alpha)\bigg|_{\alpha=0}. \end{aligned} \tag{38}$$

Here $\mathsf{A}_{\mathrm{F}}^{(ab)+}(\alpha)$ are $4M \times 4M$ antisymmetric matrices. One can write them in the form of 2×2 matrices with $2M \times 2M$ matrix entries

$$\begin{aligned} \mathsf{A}_{\mathrm{F}}^{(-+)+}(\alpha) &= \left(\begin{array}{c|c} \mathcal{A}^{(+)}_{-\beta+\mathrm{i}t} + \alpha\big(\mathrm{i}\sigma^y \otimes R^{(+)}\big) & \mathcal{G} \\ \hline -\mathcal{G}^T & \mathcal{A}^{(-)}_{-\mathrm{i}t} \end{array}\right), \\ \mathsf{A}_{\mathrm{F}}^{(+-)+}(\alpha) &= \left(\begin{array}{c|c} \mathcal{A}^{(+)}_{-\beta+\mathrm{i}t} & \mathcal{G} \\ \hline -\mathcal{G}^T & \mathcal{A}^{(-)}_{-\mathrm{i}t} + \alpha\big(\mathrm{i}\sigma^y \otimes \overline{R}^{(-)}\big) \end{array}\right), \\ \mathsf{A}_{\mathrm{F}}^{(++)+}(\alpha) &= \left(\begin{array}{c|c} \mathcal{A}^{(+)}_{-\beta+\mathrm{i}t} & \mathcal{G} - \mathrm{i}\alpha\left(\frac{I+\sigma^z}{2} \otimes K\Omega^{(-)}\right) \\ \hline -\mathcal{G}^T + \mathrm{i}\alpha\left(\frac{I+\sigma^z}{2} \otimes \Omega^{(-)}K^T\right) & \mathcal{A}^{(-)}_{-\mathrm{i}t} \end{array}\right), \\ \mathsf{A}_{\mathrm{F}}^{(--)+}(\alpha) &= \left(\begin{array}{c|c} \mathcal{A}^{(+)}_{-\beta+\mathrm{i}t} & \mathcal{G} + \mathrm{i}\alpha\left(\frac{I-\sigma^z}{2} \otimes K\Omega^{(-)}\right) \\ \hline -\mathcal{G}^T - \mathrm{i}\alpha\left(\frac{I-\sigma^z}{2} \otimes \Omega^{(-)}K^T\right) & \mathcal{A}^{(-)}_{-\mathrm{i}t} \end{array}\right) \end{aligned} \tag{39}$$

where

$$\mathcal{A}^{(\pm)}_z = \mathcal{O}^{(\pm)}\big(\mathrm{i}\sigma^y \otimes D^{(\pm)}_z\big)\mathcal{O}^{(\pm)}, \qquad \mathcal{G} = \sigma^+ \otimes F + \sigma^- \otimes \overline{H}. \tag{40}$$

The standard notations for Pauli matrices have been used to indicate the structure of the $2M \times 2M$ matrices.

The formulae (38) and (39) provide representation for the contributions in the correlators in terms of determinants of antisymmetric matrices. These formulae, however, can by viewed only as formal answers which should be simpified first in order to obtain expressions convinient for taking the thermodynamic limit. This can be achieved by reducing the determinants of $4M \times 4M$ matrices to those of $2M \times 2M$ ones by means of standard formulae for the determinants of block matrices. Taking into account that the matrices F and H have the same unitarity property (26) as the matrix L has, and the identities

$$K = FR^{(-)} = R^{(+)}H, \qquad \overline{K} = \overline{R}^{(+)}F = H\overline{R}^{(-)}, \tag{41}$$

we succeed in representing the contributions to the correlators in the following form

$$\begin{aligned} G_{\mathrm{F}}^{(\mp\pm)+} &= \frac{\partial}{\partial\alpha} \det{}^{1/2}_{2M}\Big[\mathcal{J}^{(+)}_{\frac{\beta-\mathrm{i}t}{2}}\big(I\otimes H\big)\mathcal{J}^{(-)}_{\frac{\mathrm{i}t}{2}} + \mathcal{J}^{(+)}_{-\frac{\beta-\mathrm{i}t}{2}}\big(I\otimes F\big)\mathcal{J}^{(-)}_{-\frac{\mathrm{i}t}{2}} \\ &\qquad + \alpha\,\mathcal{J}^{(+)}_{\frac{\beta-\mathrm{i}t}{2}}\Big(\tfrac{I\mp\sigma^z}{2}\otimes\overline{K}\Big)\mathcal{J}^{(-)}_{-\frac{\mathrm{i}t}{2}} + \alpha\,\mathcal{J}^{(+)}_{-\frac{\beta-\mathrm{i}t}{2}}\Big(\tfrac{I\pm\sigma^z}{2}\otimes K\Big)\mathcal{J}^{(-)}_{\frac{\mathrm{i}t}{2}}\Big]\Big|_{\alpha=0}, \\ G_{\mathrm{F}}^{(\pm\pm)+} &= \frac{\partial}{\partial\alpha} \det{}^{1/2}_{2M}\Big[\mathcal{J}^{(+)}_{\frac{\beta-\mathrm{i}t}{2}}\big(I\otimes H\big)\mathcal{J}^{(-)}_{\frac{\mathrm{i}t}{2}} + \mathcal{J}^{(+)}_{-\frac{\beta-\mathrm{i}t}{2}}\big(I\otimes F\big)\mathcal{J}^{(-)}_{-\frac{\mathrm{i}t}{2}} \\ &\qquad \mp\mathrm{i}\alpha\,\mathcal{J}^{(+)}_{\frac{\beta-\mathrm{i}t}{2}}\big(\sigma^\pm\otimes\overline{K}\big)\mathcal{J}^{(-)}_{-\frac{\mathrm{i}t}{2}} \mp\mathrm{i}\alpha\,\mathcal{J}^{(+)}_{-\frac{\beta-\mathrm{i}t}{2}}\big(\sigma^\pm\otimes K\big)\mathcal{J}^{(-)}_{\frac{\mathrm{i}t}{2}}\Big]\Big|_{\alpha=0}. \end{aligned} \tag{42}$$

Here the $2M \times 2M$ matrices $\mathcal{J}^{(\pm)}_z$ depend on the variable z pure exponentially:

$$\mathcal{J}^{(\pm)}_z = \exp z\mathcal{E}^{(\pm)}, \qquad \mathcal{E}^{(\pm)} = \sigma^z\otimes\varepsilon^{(\pm)} + \sigma^x\otimes\varkappa^{(\pm)} \tag{43}$$

where $\varepsilon^{(\pm)}$ and $\varkappa^{(\pm)}$ are $M\times M$ diagonal matrices with entries

$$(\varepsilon^{(\pm)})_{ab} = \varepsilon(p_a)\delta_{ab}, \qquad (\varkappa^{(\pm)})_{ab} = \varkappa(p_a)\delta_{ab}, \qquad p\in\mathbb{X}^\pm. \tag{44}$$

The matrices $\mathcal{E}^{(\pm)}$ can be regarded as the lattice Dirac "operators" in the momentum representation which correspond to the Hamiltonians (7). Remarkably, though Dirac operators do not enter into the matrix elements of the operators $\exp(-z\mathbf{H}^\pm)$, they arise explicitly in the final expressions for the contributions in the correlators.

The contributions $G_{\mathrm{F}}^{(ab)-}$ and $G_{\mathrm{B}}^{(ab)\pm}$ admit representations similar to (42) for $G_{\mathrm{F}}^{(ab)+}$. The formulae for $G_{\mathrm{F}}^{(ab)-}$ can be obtained from (42) by interchanging sets of the momenta $\mathbb{X}^+ \leftrightarrow \mathbb{X}^-$. The representations for the contributions $G_{\mathrm{B}}^{(ab)\pm}$ concide with those for $G_{\mathrm{F}}^{(ab)\pm}$ with the difference in signs of some terms in the determinant. The complete list of the results as well as details of derivation of the representations like (42) will be given elsewhere.

V. CORRELATORS IN THE THERMODYNAMIC LIMIT

The correlation functions (12) are most interesting in the thermodynamic limit, as M tends to infinity. In this limit all contributions $G^{(ab)\pm}_{F,B}$ and $Z^{\pm}_{F,B}$ are singular but the correlators (13) remain finite. It is known [1–4], and this can be established directly from (15) and the results of the previous Section, that in the thermodynamic limit the "fermionic" contributions tend to each other and prevail over the "bosonic" ones both in the denominator and numerator of (13). Thus, for the correlators in the thermodynamic limit one has

$$\langle\!\langle \sigma^a_{m+1}(t)\,\sigma^b_1(0)\rangle\!\rangle = \lim_{M\to\infty} \langle\!\langle \sigma^a_{m+1}(t)\,\sigma^b_1(0)\rangle\!\rangle_M = \lim_{M\to\infty} \frac{G^{(ab)+}_F}{Z^+_F}. \tag{45}$$

In the thermodynamic limit, due to the formulae (42) and (45), we obtain expressions for the correlators in terms of Fredholm determinants of linear integral operators with 2×2 matrix kernels defined on the strip $[-\pi,\pi]$. It is convenient to write these formulae in terms of integral operators defined on the unit circle acting on vector-functions by the rule

$$\big[\hat{A}f\big]_j(\lambda) = \oint\limits_{|\mu|=1} \sum_{k=1,2} (A(\lambda,\mu))_{jk} f_k(\mu)\,\mathrm{d}\mu\,, \qquad \lambda\in S^1. \tag{46}$$

Here the matrix $A(\lambda,\mu)$ with the entries $(A(\lambda,\mu))_{jk}$ is the kernel of the integral operator $\hat{A}$.

For the correlators in the thermodynamic limit we obtain that the following Fredholm determinant representations are valid

$$\begin{aligned}
\langle\!\langle \sigma^-_{m+1}(t)\,\sigma^+_1(0)\rangle\!\rangle &= \left(\frac{\mathrm{d}}{\mathrm{d}\alpha} + \frac{(G)_{22}}{2}\right) \det{}^{1/2}\left[\hat{I}+\hat{V}+\frac{\alpha}{2}\hat{W}^{(11)}_{--} - \frac{\alpha}{2}\hat{W}^{(22)}_{++} + \mathrm{i}\alpha\hat{W}^{(22)}_{+-}\right]\Bigg|_{\alpha=0},\\
\langle\!\langle \sigma^+_{m+1}(t)\,\sigma^-_1(0)\rangle\!\rangle &= \left(\frac{\mathrm{d}}{\mathrm{d}\alpha} + \frac{(G)_{11}}{2}\right) \det{}^{1/2}\left[\hat{I}+\hat{V}+\frac{\alpha}{2}\hat{W}^{(22)}_{--} - \frac{\alpha}{2}\hat{W}^{(11)}_{++} + \mathrm{i}\alpha\hat{W}^{(11)}_{+-}\right]\Bigg|_{\alpha=0},\\
\langle\!\langle \sigma^+_{m+1}(t)\,\sigma^+_1(0)\rangle\!\rangle &= \left(\frac{\mathrm{d}}{\mathrm{d}\alpha} + \frac{(G)_{21}}{2\mathrm{i}}\right) \det{}^{1/2}\left[\hat{I}+\hat{V}+\frac{\alpha}{2\mathrm{i}}\hat{W}^{(12)}_{--} - \frac{\alpha}{2\mathrm{i}}\hat{W}^{(12)}_{++} + \alpha\hat{W}^{(12)}_{+-}\right]\Bigg|_{\alpha=0},\\
\langle\!\langle \sigma^-_{m+1}(t)\,\sigma^-_1(0)\rangle\!\rangle &= \left(\frac{\mathrm{d}}{\mathrm{d}\alpha} - \frac{(G)_{12}}{2\mathrm{i}}\right) \det{}^{1/2}\left[\hat{I}+\hat{V}-\frac{\alpha}{2\mathrm{i}}\hat{W}^{(21)}_{--} + \frac{\alpha}{2\mathrm{i}}\hat{W}^{(21)}_{++} - \alpha\hat{W}^{(21)}_{+-}\right]\Bigg|_{\alpha=0}.
\end{aligned} \tag{47}$$

Here $\hat{I}$ is the unity operator and $\hat{V}$ is an integrable integral operator with the kernel

$$V(\lambda,\mu) = \frac{E_+(\lambda)\big[E_-(\mu)\big]^T - E_-(\lambda)\big[E_+(\mu)\big]^T}{\lambda-\mu}. \tag{48}$$

The kernels of the operators $\hat{W}^{(jk)}_{--}$, $\hat{W}^{(jk)}_{++}$ and $\hat{W}^{(jk)}_{+-}$ are

$$W^{(jk)}_{\pm\pm}(\lambda,\mu) = \frac{E_\pm(\lambda)\,e^{(jk)}\big[E_\pm(\mu)\big]^T}{\mathrm{i}\mu}, \qquad W^{(jk)}_{+-}(\lambda,\mu) = \frac{E_+(\lambda)\,e^{(jk)}\,G\big[E_-(\mu)\big]^T}{\mathrm{i}\mu} \tag{49}$$

where $e^{(jk)}$ are standard 2×2 basis matrices, $e^{(11)} = (I+\sigma^z)/2$, $e^{(22)} = (I-\sigma^z)/2$, $e^{(12)} = \sigma^+$ and $e^{(21)} = \sigma^-$. The matrix-functions $E_\pm(\lambda) = E_\pm(\lambda; m, t, h, \gamma, \beta)$ determining the kernels of the operators are

$$E_+(\lambda) = E_-(\lambda)\, E(\lambda)\,, \qquad E_-(\lambda) = \left[\frac{\Theta(\lambda)}{\pi}\right]^{1/2} \exp\bigl(\mathrm{i}t\Phi(\lambda)/2\bigr)\, \lambda^{-m/2}, \tag{50}$$

and the matrix-functions $E(\lambda) = E(\lambda; m, t, h, \gamma)$ and $G = G(m, t, h, \gamma)$ are defined follows

$$E(\lambda) = \frac{1}{\pi}\mathrm{P.v.} \oint\limits_{|\mu|=1} \frac{\exp\bigl(-\mathrm{i}t\Phi(\mu)\bigr)\, \mu^m}{\mu - \lambda}\, \mathrm{d}\mu\,, \qquad G = \frac{1}{2\pi\mathrm{i}} \oint\limits_{|\mu|=1} \exp\bigl(-\mathrm{i}t\Phi(\mu)\bigr)\, \mu^{m-1}\, \mathrm{d}\mu. \tag{51}$$

At last, the matrices $\Phi(\lambda) = \Phi(\lambda; h, \gamma)$ and $\Theta(\lambda) = \Theta(\lambda; h, \gamma, \beta)$, the latter playing the role of the Fermi weight, are

$$\Phi(\lambda) = (h - \lambda - \lambda^{-1})\, \sigma^z - \mathrm{i}\gamma(\lambda - \lambda^{-1})\, \sigma^x\,, \qquad \Theta(\lambda) = \frac{1}{1 + \exp\bigl(\beta\Phi(\lambda)\bigr)}. \tag{52}$$

It is worth to note that since the matrix $\Phi(\lambda)$ is symmetric the same holds for the matrices $\Theta(\lambda)$, $E(\lambda)$, $E_-(\lambda)$ and G. In particular, this leads to the fact that the kernel of the operator $\hat{V}$ given by Eqn. (48) is non-singular function of the variables λ and μ.

Evaluation of the derivatives with respect to α is straightforward. Introducing the matrix-functions $F_\pm(\lambda)$ as the solutions of the integral equations

$$\left[(\hat{I} + \hat{V})F_\pm\right](\lambda) = E_\pm(\lambda) \tag{53}$$

one can define the so-called "big" potentials [7], which are 2×2 matrices in the case considered, as follows

$$B_{jk} = \frac{1}{\mathrm{i}} \oint\limits_{|\mu|=1} \bigl[F_j(\mu)\bigr]^T E_k(\mu)\, \frac{\mathrm{d}\mu}{\mu}, \qquad (j, k = \pm). \tag{54}$$

Then, in terms of the so-called "small" potentials which are, in turn, 2×2 matrices defined similarly to the isotropic case [10] by the relations

$$b_{++} = B_{++} - 2\mathrm{i}B_{+-}\, G - 2G\,, \qquad b_{--} = B_{--} \tag{55}$$

we obtain finally for the correlators the expressions

$$\begin{aligned}
\bigl\langle\!\bigl\langle \sigma^-_{m+1}(t)\, \sigma^+_1(0) \bigr\rangle\!\bigr\rangle &= \frac{1}{4}\, \det{}^{1/2}\Bigl(\hat{I} + \hat{V}\Bigr)\Bigl[(b_{--})_{11} - (b_{++})_{22}\Bigr], \\
\bigl\langle\!\bigl\langle \sigma^+_{m+1}(t)\, \sigma^-_1(0) \bigr\rangle\!\bigr\rangle &= \frac{1}{4}\, \det{}^{1/2}\Bigl(\hat{I} + \hat{V}\Bigr)\Bigl[(b_{--})_{22} - (b_{++})_{11}\Bigr], \\
\bigl\langle\!\bigl\langle \sigma^+_{m+1}(t)\, \sigma^+_1(0) \bigr\rangle\!\bigr\rangle &= \frac{1}{4\mathrm{i}} \det{}^{1/2}\Bigl(\hat{I} + \hat{V}\Bigr)\Bigl[(b_{--})_{12} - (b_{++})_{12}\Bigr], \\
\bigl\langle\!\bigl\langle \sigma^-_{m+1}(t)\, \sigma^-_1(0) \bigr\rangle\!\bigr\rangle &= \frac{\mathrm{i}}{4}\, \det{}^{1/2}\Bigl(\hat{I} + \hat{V}\Bigr)\Bigl[(b_{--})_{21} - (b_{++})_{21}\Bigr].
\end{aligned} \tag{56}$$

Here $(b_{\pm\pm})_{ab}$ with $a, b = 1, 2$ denote entries of the matrices (55). Our preliminary analysis of the Reimann-Hilbert problem associated with the integral operator $\hat{V}$ shows that among the eight functions $(b_{\pm\pm})_{ab}$ only three of them are independent. The matrices (55) have the structure

$$b_{--} = \begin{pmatrix} g_- & \mathrm{i}g_0 \\ -\mathrm{i}g_0 & -g_+ \end{pmatrix}, \qquad b_{++} = \begin{pmatrix} g_+ & -\mathrm{i}g_0 \\ \mathrm{i}g_0 & -g_- \end{pmatrix} \tag{57}$$

where g_+, g_- and g_0 are some functions.

In conclusion, we have considered the problem of exact calculation of the temperature time-dependent two-point correlators of the "odd" spin operators in the anisotropic XY spin chain. Basing on the formulae (42) we have obtained for the correlators in the thermodynamic limit the Fredholm determinant representations (47) and (56) which are our main results. The matrix-functions entering the kernels of the operators possess essentially the same structure as in the isotropic case [9]. This point is important for direct application of the standard methods [7, 10] for evaluation of derivatives of the Fredholm determinant with respect to time and distance, and derivation of the non-linear differential equations satesfied by the functions g_+, g_- and g_0. The essential tool in the further study is the associated Riemann-Hilbert problem. This will be a subject of our future publications.

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