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Temperature correlators in the one-dimensional Hubbard model in the strong coupling limit

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ABSTRACT

We consider the one-dimensional Hubbard model with the infinitely strong repulsion. The two-point dynamical temperature correlation functions are calculated. They are represented as Fredholm determinants of linear integrable integral operators.

1 Introduction

The one-dimensional Hubbard model [1] is one of the most interesting and important model of strongly correlated electrons (see, e.g., book [2]). The hamiltonian of the model with the chemical potential h in the external constant field B describes interacting fermions on the one-dimensional periodic lattice of length L:

$$H_U = -\sum_{x=1}^{L}\sum_{\alpha=\uparrow,\downarrow}\left(c^\dagger_{x,\alpha}c_{x-1,\alpha}+c^\dagger_{x-1,\alpha}c_{x,\alpha}\right) \\ +h\sum_{x=1}^{L}(n_{x,\uparrow}+n_{x,\downarrow})-B\sum_{x=1}^{L}(n_{x,\uparrow}-n_{x,\downarrow})+U\sum_{x=1}^{L}n_{x,\uparrow}n_{x,\downarrow}. \quad (1)$$

Here $c_{x,\alpha}$ and $c^\dagger_{x,\alpha}$ ($x=1,\dots,L$; $\alpha=\uparrow,\downarrow$; $c_{0,\alpha}\equiv c_{L,\alpha}$; $c^\dagger_{0,\alpha}\equiv c^\dagger_{L,\alpha}$) are canonical fermion fields with the anticommutation relations

$$[c_{x,\alpha},c^\dagger_{y,\beta}]_+=\delta_{x,y}\delta_{\alpha,\beta} \quad (2)$$

The density operators of electrons with spin up ($\alpha=\uparrow$) and down ($\alpha=\downarrow$) are $n_{x,\alpha}=c^\dagger_{x,\alpha}c_{x,\alpha}$. The solution of this model by means of the two-component (nested) Bethe Anzatz [3, 4] was given in [5].

The temperature correlation functions are defined in a standard way as temperature normalized mean values. In the present paper we consider two-point correlation functions of canonical field operators,

$$\langle c^\dagger_{x,\alpha}(t)c_{0,\alpha}(0)\rangle^{(T,L)}=\frac{\mathrm{Sp}\left[e^{-H_U/T}c^\dagger_{x,\alpha}(t)c_{0,\alpha}(0)\right]}{\mathrm{Sp}\left[e^{-H_U/T}\right]},$$

$$\langle c_{x,\alpha}(t)c^\dagger_{0,\alpha}(0)\rangle^{(T,L)}=\frac{\mathrm{Sp}\left[e^{-H_U/T}c_{x,\alpha}(t)c^\dagger_{0,\alpha}(0)\right]}{\mathrm{Sp}\left[e^{-H_U/T}\right]}, \quad (3)$$

and of density operators,

$$\langle n_{x,\alpha}(t)n_{0,\beta}(0)\rangle^{(T,L)}=\frac{\mathrm{Sp}\left[e^{-H_U/T}n_{x,\alpha}(t)n_{0,\beta}(0)\right]}{\mathrm{Sp}\left[e^{-H_U/T}\right]}, \quad (4)$$

where $\alpha,\beta=\uparrow,\downarrow$. The dependence on time is introduced as usual,

$$c_{x,\alpha}(t)=e^{iH_Ut}c_{x,\alpha}e^{-iH_Ut},\quad c^\dagger_{x,\alpha}(t)=e^{iH_Ut}c^\dagger_{x,\alpha}e^{-iH_Ut},$$
$$n_{x,\alpha}(t)=e^{iH_Ut}n_{x,\alpha}e^{-iH_Ut}. \quad (5)$$

The trace Sp is taken in the Fock space $\mathcal{F}$ of dimension 4^L where the hamiltonial of the Hubbard model acts. The correlation functions are mostly interesting in the thermodynamic limit ($L\to\infty$; h and B are kept fixed):

$$\langle\cdots\rangle^{(T)}=\lim_{L\to\infty}\langle\cdots\rangle^{(T,L)}. \quad (6)$$

Our aim is to present the results on calculation of the correlation functions in the Hubbard model in the strong coupling limit, $U \to +\infty$. The two-point temperature correlation functions are represented as Fredholm determinants of linear integral operators of a special kind (the "integrable" integral operators, in the sence of the paper [6]).

It is to be mentioned that the recent progress in calculating correlation functions of quantum solvable models is based on the fact that they are governed by classical integrable differential equations. That the language of classical differential equations is quite natural for the description of quantum correlation functions was realized a time ago [7, 8, 9, 10]. The idea of the approach suggested in [11, 12, 6] is to consider the Fredholm determinant in the representation for a correlation function of a quantum integrable model as a tau-function for a classical integrable system (see also the book [13] where the results for the simplest model of one-dimensional impenetrable bosons are reviewed). The necessary first step (which is also of interest by itself) in this approach is to represent the correlation function as the Fredholm determinant of a linear "integrable" integral operator. The first determinant representation of this kind was given in [14, 15] for the equal-time temperature correlators of the one-dimentional impenetrable bosons. For the two-component one-dimensional impenetrable Bose and Fermi gases the representations of this type were obtained recently in papers [16, 17].

The Hubbard model is a lattice analogue of the two-component Fermi gas. It should be noted that the physics of the Hubbard model at $U = +\infty$ has some interesting properties (see, e.g., [18]). In particular, the ground state becomes degenerate at zero external field, i.e., the point $B = 0$ is the point of the phase transition (the same take place in the Fermi gas as well).

We use the technique of calculating temperature correlation functions of exactly solvable quantum two-component models with infinitely strong coupling developed in papers [16, 17] for one-dimensional two-component Bose and Fermi gases. The starting point of the approach is using the eigenfunctions of the $XX0$ spin chain with the periodic boundary conditions to solve the auxiliarly lattice problem of the nested Bethe Ansatz. This gives an explicit expression for the eigenstates of the two-component models at the point of infinite coupling.

2 The hamiltonian and eigenstates at $U = \infty$

In the strong coupling limit, the states with double occupancies (i.e., the states with a pair of electrons at least at one cite of the lattice) have infinite energy and therefore are absent in the physical space of states. The dynamics of the Hubbard model in this limit is described by an effective hamiltonian which can

be written in the form [19, 20]

$$H = \mathcal{P}\Bigg[-\sum_{x=1}^{L}\sum_{\alpha=\uparrow,\downarrow}\left(c^{\dagger}_{x,\alpha}c_{x-1,\alpha}+c^{\dagger}_{x-1,\alpha}c_{x,\alpha}\right) + h\sum_{x=1}^{L}(n_{x,\uparrow}+n_{x,\downarrow}) - B\sum_{x=1}^{L}(n_{x,\uparrow}-n_{x,\downarrow})\Bigg]\mathcal{P}, \tag{7}$$

where

$$\mathcal{P} = \prod_{x=1}^{L}(1-n_{x,\uparrow}n_{x,\downarrow}). \tag{8}$$

The projector $\mathcal{P}$ extracts the physical space of states $\mathcal{H}$ of dimention 3^L from the space $\mathcal{F}$ (of dimention 4^L) in which the canonical Hubbard operators act.

Eigenstates of the effective hamiltonian H (all belonging to the space $\mathcal{H}$) can be constructed analogously to the case of the two-component impenetrable Fermi gas [16, 17]. The eigenstates have the form [5]

$$|\Psi_{N,M}(k;\lambda)\rangle = \sum_{z_1,\ldots,z_N=1}^{L}\sum_{\alpha_1,\ldots,\alpha_N=\uparrow,\downarrow}\chi_{N,M}^{\alpha_1\ldots\alpha_N}(z_1,\ldots,z_N|k;\lambda) \; c^{\dagger}_{z_1,\alpha_1}\ldots c^{\dagger}_{z_N,\alpha_N}|0\rangle, \tag{9}$$

where the Fock vacuum $|0\rangle$ is defined as usual, $c_{z,\alpha}|0\rangle = 0$, $\langle 0|c^{\dagger}_{z,\alpha} = 0$, $\langle 0|0\rangle = 1$. The eigenstates are parametrized by set of N unequal real numbers (quasimomenta of N particles), $k \equiv k_1,\ldots,k_N$, and the set of M unequal real numbers (quasimomenta of the auxiliarly lattice problem), $\lambda \equiv \lambda_1,\ldots,\lambda_M$. The number N is the total number of electrons in the state while the number M is the number of electrons with spin down. Thus M of N values α in (9) are $\downarrow$, and $N-M$ are $\uparrow$. The momenta k and momenta λ are not arbitrary but must obey the system of the Bethe equations (the nested Bethe Ansatz) [3, 4, 5]. In the case $U = +\infty$ using the $XX0$ basis for the auxiliarly problem results in the simplified form of the Bethe equations,

$$\begin{aligned} e^{ik_aL} &= e^{i\Lambda}, \qquad a = 1,\ldots,N, \\ e^{i\lambda_bN} &= (-1)^{M+1}, \qquad b = 1,\ldots,M, \end{aligned} \tag{10}$$

where

$$\Lambda \equiv \sum_{b=1}^{M}\lambda_b. \tag{11}$$

This system can be solved explicitly; the equations are separated by the substitution

$$k_a = \tilde{k}_a + \frac{\Lambda}{L}. \tag{12}$$

The permitted values of the momenta are

$$(\tilde{k}_a)_j = \frac{2\pi}{L} j, \qquad j = 0, \ldots, L-1,$$
$$(\lambda_b)_l = \frac{2\pi}{N}\left(-\frac{N}{2} + \frac{1+(-1)^{N-M}}{4} + l\right), \qquad l = 0, \ldots, N-1. \tag{13}$$

The wave function $\chi_{N,M}$ is

$$\chi_{N,M}^{\alpha_1 \ldots \alpha_N}(z_1, \ldots, z_N | k; \lambda)$$
$$= \frac{1}{N!}\left[\sum_P \xi_{N,M}^{\alpha_{P_1} \ldots \alpha_{P_N}}(\lambda)\theta(z_{P_1} < \ldots < z_{P_N})\right] \det_N\{e^{ik_a z_b}\}. \tag{14}$$

The sum here is taken over the permutations of N numbers, $P : (1, \ldots, N) \to (P_1, \ldots, P_N)$. The function $\theta(z_1 < \ldots < z_N)$ is equal to 1 if $z_1 < \ldots < z_N$ and is equal to zero otherwise. The determinant $\det_N\{e^{ik_a z_b}\}$ denotes the determinant of the $N \times N$ matrix with elements $e^{ik_a z_b}$. The spin part of the wave function is described by components of the 2^N-dimensional vector $|\xi_{N,M}(\lambda))$. As in the case of the two-component impenetrable gas, we use the vectors $|\xi_{N,M}(\lambda))$ to be eigenvectors of the hamiltonian of the $XX0$ spin chain in the form given in [21]:

$$|\xi_{N,M}(\lambda)) = \sum_{n_1, \ldots, n_M = 1}^{N} \varphi_{N,M}(n_1, \ldots, n_M | \lambda)\ \sigma_{(n_1)}^- \ldots \sigma_{(n_M)}^- | \Uparrow_N), \tag{15}$$

with the wave function

$$\varphi_{N,M}(n_1, \ldots, n_M | \lambda) = \frac{1}{M!}\left[\prod_{1 \le j < l \le N} \mathrm{sgn}(n_l - n_j)\right] \det_M\{e^{i\lambda_a n_b}\}. \tag{16}$$

Pauli matrices are defined as usual,

$$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{17}$$

The vacuum $|\Uparrow_N)$ of the spin chain with N sites is defined as $\sigma_{(n)}^+ |\Uparrow_N) = 0$, $(\Uparrow_N | \sigma_{(n)}^- = 0$, $(\Uparrow_N | \Uparrow_N) = 1$, i.e.,

$$|\Uparrow_N) = \otimes_{n=1}^N \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{(n)}. \tag{18}$$

Eigenvalues of the hamiltonian (7) on the eigenstates (9),

$$H\ |\Psi_{N,M}(k, \lambda)\rangle = E_{N,M}(k)\ |\Psi_{N,M}(k, \lambda)\rangle, \tag{19}$$

are

$$E_{N,M}(k) = \sum_{a=1}^{N} \varepsilon(k_a) - hN + B(N-2M), \tag{20}$$

where $\varepsilon(k)$ is one-particle dispersion

$$\varepsilon(k_a) = -2\cos k_a = -2\cos\left(\tilde{k}_a + \frac{\Lambda}{L}\right). \tag{21}$$

Thus, the eigenenergies depend on $\lambda_1, \ldots, \lambda_M$ via their sum, Λ. It is used essensially in our approach.

The states $|\Psi_{N,M}(k;\lambda)\rangle$ form a complete orthogonal set in the physical space $\mathcal{H}$ (of dimention 3^L); the normalization is

$$\langle\Psi_{N,M}(k;\lambda)|\Psi_{N,M}(k;\lambda)\rangle = L^N N^M. \tag{22}$$

The effect of the infinitely strong interaction consists in contraction of observables (quantum operators) onto the space $\mathcal{H}$. The unit operator in $\mathcal{H}$ is just the projector $\mathcal{P}$ in $\mathcal{F}$. Due to the completness, this operator possesses the decomposition

$$\mathcal{P} = \sum_{\text{states}} |\Psi_{N,M}(k;\lambda)\rangle \frac{1}{L^N N^M} \langle\Psi_{N,M}(k;\lambda)|, \tag{23}$$

where the summation is performed over all 3^L basis states in $\mathcal{H}$,

$$\sum_{\text{states}} \equiv \sum_{N=0}^{L} \sum_{M=0}^{N} \sum_{\substack{k_1<\ldots<k_N \\ \lambda_1<\ldots<\lambda_M}} . \tag{24}$$

Due to (23), the effective hamiltonian H (7) can be also defined by its spectral decomposition

$$H = \sum_{\text{states}} |\Psi_{N,M}(k;\lambda)\rangle \frac{E_{N,M}(k)}{L^N N^M} \langle\Psi_{N,M}(k;\lambda)|. \tag{25}$$

The temperature correlation functions in the strong coupling limit acquire the form:

$$\langle c^\dagger_{x,\alpha}(t) c_{0,\alpha}(0)\rangle^{(T,L)} = \frac{\mathrm{Tr}\left[e^{-H/T}\tilde{c}^\dagger_{x,\alpha}(t)\tilde{c}_{0,\alpha}(0)\right]}{\mathrm{Tr}\left[e^{-H/T}\right]},$$

$$\langle c_{x,\alpha}(t) c^\dagger_{0,\alpha}(0)\rangle^{(T,L)} = \frac{\mathrm{Tr}\left[e^{-H/T}\tilde{c}_{x,\alpha}(t)\tilde{c}^\dagger_{0,\alpha}(0)\right]}{\mathrm{Tr}\left[e^{-H/T}\right]},$$

$$\langle n_{x,\alpha}(t) n_{0,\beta}(0)\rangle^{(T,L)} = \frac{\mathrm{Tr}\left[e^{-H/T}\tilde{n}_{x,\alpha}(t)\tilde{n}_{0,\beta}(0)\right]}{\mathrm{Tr}\left[e^{-H/T}\right]}. \tag{26}$$

Here the trace Tr (which is different from Sp in (3) and (4)), is to be taken in the space $\mathcal{H}$, i.e., by definition,

$$\mathrm{Tr}\Big[\cdots\Big] = \sum_{\text{states}} \frac{1}{L^N N^M} \langle \Psi_{N,M}(k;\lambda)| \cdots |\Psi_{N,M}(k;\lambda)\rangle. \tag{27}$$

The tildes over the Hubbard operators in the numerators in (26) mean the contraction of these operators onto the space $\mathcal{H}$, i.e.,

$$\tilde{c}_{x,\alpha} \equiv \mathcal{P} c_{x,\alpha} \mathcal{P}, \qquad \tilde{c}^{\dagger}_{x,\alpha} \equiv \mathcal{P} c^{\dagger}_{x,\alpha} \mathcal{P}, \qquad \tilde{n}_{x,\alpha} \equiv \mathcal{P} n_{x,\alpha} \mathcal{P}. \tag{28}$$

The time dependence is described by the relations similar to (5) with the effective hamiltonian H given by (7). The equations (26) exhibit the recipe for calculation of the correlators in the strong coupling limit. Indeed, all states belonging to the subspace $\mathcal{F}\backslash\mathcal{H}$ (of dimention $4^L - 3^L$) in the decomposition of the unit operator in the Fock space $\mathcal{F}$ inserted between canonical field operators in the numerators in (3) do not contribute to the correlators due to infinitely strong oscilation for any small (but finite) value of t as U tends to infinity. The same happens with the correlators of density operators, but since $[\mathcal{P}, n_{x,\alpha}(t)] = 0$ such intermediate states do not contribute already due to the temperature exponential. Thus, equations (26) together with (28) and (23) allow one to express the correlators in the strong coupling limit through matrix elements (form factors) of Hubbard operators $c_{x,\alpha}(t)$, $c^{\dagger}_{x,\alpha}(t)$, $n_{x,\alpha}(t)$ between two states from $\mathcal{H}$ only. It is to be emphasized that equations (26) will produce the correct answers also for the equal-time correlation functions at $U = \infty$ (since t should be put equal to zero after taking the limit $U \to \infty$).

The equal-time correlators of the canonical field operators (given by the first and the second equations in (26) at $t = 0$) should satisfy simple relations governed by the anticommutation relations between $\tilde{c}_{x,\alpha}$ and $\tilde{c}^{\dagger}_{y,\beta}$. If $x \neq y$ then $\tilde{c}_{x,\alpha}$ and $\tilde{c}^{\dagger}_{y,\beta}$ anticommute, while at $x = y$ and $\alpha = \beta$ the anticommutators are different from (2) being equal to

$$[\tilde{c}_{x,\uparrow}, \tilde{c}^{\dagger}_{x,\uparrow}]_+ = \mathcal{P} - \tilde{n}_{x,\downarrow}, \qquad [\tilde{c}_{x,\downarrow}, \tilde{c}^{\dagger}_{x,\downarrow}]_+ = \mathcal{P} - \tilde{n}_{x,\uparrow}. \tag{29}$$

In particular, it follows from these relations that

$$\tilde{c}_{x,\uparrow} \tilde{c}^{\dagger}_{x,\uparrow} = \mathcal{P} - (\tilde{n}_{x,\uparrow} + \tilde{n}_{x,\downarrow}) \tag{30}$$

(and the same for the $\tilde{c}_{x,\downarrow} \tilde{c}^{\dagger}_{x,\downarrow}$). Having in mind that $\tilde{c}^{\dagger}_{x,\alpha} \tilde{c}_{x,\alpha} = \tilde{n}_{x,\alpha}$, and that $\mathcal{P}$ play the role of the unit operator in the space $\mathcal{H}$ it is easy to see what the relations between correlators at $t = 0$, $x = 0$ are. Let us note that these relations are different from those which one gets from (3) and (2). It means that the limits $U \to \infty$ and $t \to 0$ do not "commute".

The operators $\tilde{c}_{x,\alpha}$ and $\tilde{c}^{\dagger}_{x,\alpha}$, due to (23), can be also defined by means of their decompositions, e.g., one has

$$\tilde{c}_{x,\uparrow} = \sum_{N=0}^{L-1}\sum_{M=0}^{N}\sum_{\substack{q_1<\ldots<q_N\\ \mu_1<\ldots<\mu_M}}\sum_{\substack{k_1<\ldots<k_{N+1}\\ \lambda_1<\ldots<\lambda_M}} |\Psi_{N,M}(q;\mu)\rangle \frac{\mathcal{F}^{(N+1,M)}_{x,\uparrow}(q;\mu|k;\lambda)}{L^{2N+1}N^M(N+1)^M}\langle\Psi_{N+1,M}(k;\lambda)|, \tag{31}$$

where $\mathcal{F}^{(N+1,M)}_{x,\uparrow}(q;\mu|k;\lambda)$ are matrix elements (form factors) of the operator $c_{x,\uparrow}$ between two states from $\mathcal{H}$:

$$\mathcal{F}^{(N+1,M)}_{x,\uparrow}(q;\mu|k;\lambda) \equiv \langle\Psi_{N,M}(q;\mu)|c_{x,\uparrow}|\Psi_{N+1,M}(k;\lambda)\rangle. \tag{32}$$

Explicit calculation results in the following representation

$$\mathcal{F}^{(N+1,M)}_{x,\uparrow}(q;\mu|k;\lambda) = e^{-i\left(\frac{\Lambda-\Theta}{2}\right)N}\det\nolimits_M W \det\nolimits_{N+1} D \exp\left\{ix\sum_{a=1}^{N+1}k_a - ix\sum_{b=1}^{N}q_b\right\}, \tag{33}$$

where $\Lambda = \sum_{a=1}^{M}\lambda_a$ and $\Theta = \sum_{b=1}^{M}\mu_b$. Elements of the $(N+1)\times(N+1)$ matrix D are $(a = 1,\ldots,N+1;\ b = 1,\ldots,N)$

$$(D)_{ab} = \sin\Big(\frac{\Lambda-\Theta}{2}\Big)\cot\Big(\frac{k_a-q_b}{2}\Big), \qquad (D)_{a,N+1} = 1, \tag{34}$$

and elements of the $M\times M$ matrix W are $(a,b = 1,\ldots,N)$

$$(W)_{ab} = \sum_{n=1}^{N} e^{in(\lambda_a-\mu_b)}. \tag{35}$$

The similar representation can be obtained for the form factors of operator $c_{x,\downarrow}$. The form factors of operators $c^{\dagger}_{x,\alpha}$ can be obtained from the form factors of operators $c_{x,\alpha}$ by means of complex conjugation. The form factors of density operators $n_{x,\alpha}$ can be also calculated; the corresponding results are given in our recent paper [22].

The representations for the form factors (33) allow us to calculate the temperature correlation functions in the thermodynamic limit.

3 Correlation functions in the thermodynamic limit

In this Section the results of calculation of the two-point temperature correlation functions in the Hubbard model at $U = \infty$ in the thermodynamic limit are given.

To derive the representations below we calculate first the correlators on the finite lattice (26) and go to the thermodynamic limit (6), $L \to \infty$, after that. On this way we use the technique of papers [16, 17].

In the thermodynamic limit the correlation functions considered are represented as Fredholm determinants of linear integral operators. The integral operators which enter the representations for the correlators act on arbitrary function $f(k)$ according to the rule:

$$(\hat{\mathcal{A}} \cdot f)(k) = \int_{-\pi}^{\pi} dk' \, \mathcal{A}(k,k') f(k'), \tag{36}$$

where the function $\mathcal{A}(k,k')$ is the kernel of the integral operator $\hat{\mathcal{A}}$.

In order to write down the representations for the correlators, let us introduce some notations. Define the functions

$$E(k) = E(k;x,t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dq \frac{e^{-it\varepsilon(q)+ixq} - e^{-it\varepsilon(k)+ixk}}{\tan(\frac{q-k}{2})},$$

$$G = G(x,t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dq e^{-it\varepsilon(q)+ixq}. \tag{37}$$

The distance x is an arbitrary integer; the dispersion is $\varepsilon(q) = -2\cos q$ (see (21)) where the momenta q and k in (37) take any values on the interval $[-\pi, \pi]$. In particular, $G(x,t) = i^x J_x(2t)$ where J_x is the Bessel function ($G(x,0) = \delta_{x,0}$).

The following pair of functions play an important role:

$$\ell_+(\eta|k) = \left\{ \frac{1-\cos\eta}{2} E_+(k) + \frac{\sin\eta}{2} \frac{1}{E_-(k)} \right\} \sqrt{\vartheta(k)},$$

$$\ell_-(k) = E_-(k)\sqrt{\vartheta(k)}, \tag{38}$$

where $\eta \in [-\pi, \pi]$ is a parameter and $\vartheta(k)$ is the Fermi weight,

$$\vartheta(k) = \frac{e^{-B/T}}{\cosh\frac{B}{T} + e^{(\varepsilon(k)-h)/T}}. \tag{39}$$

Here h is the chemical potential and B is the external constant field. The functions $E_\pm(k)$ are

$$E_+(k) = E(k)\, E_-(k), \qquad E_-(k) = \exp\left(\frac{it\varepsilon(k) - ixk}{2}\right). \tag{40}$$

We introduce also the function

$$F(\gamma;\eta) := 1 + \sum_{p=1}^{\infty} \gamma^{-p} (e^{ip\eta} + e^{-ip\eta}), \tag{41}$$

where

$$\gamma = 1 + e^{2B/T}. \tag{42}$$

It is worth mentioning that $\gamma \in [1, \infty)$ for any real external field B. At the point $\gamma = 1$ ($B = -\infty$) one has $F(1;\eta) = 2\pi\Delta(\eta)$ where $\Delta(\eta)$ is the 2π-periodic delta-function.

Now we are ready to formulate the main results.

For the correlation function of the canonical field operators of the Hubbard model at $U = \infty$ we obtain the following representations:

$$\begin{aligned}
\langle c^{\dagger}_{x,\uparrow}(t) c_{0,\uparrow}(0)\rangle^{(T)} &= e^{-it(h-B)} \frac{1}{2\pi} \int\limits_{-\pi}^{\pi} d\eta\, F(\gamma;\eta) \\
&\left[\det\left(\hat{\mathcal{I}} + \gamma\, \hat{\mathcal{Q}}(\eta) + \hat{\mathcal{R}}^{(-)}\right) - \det\left(\hat{\mathcal{I}} + \gamma\, \hat{\mathcal{Q}}(\eta)\right)\right], \\
\langle c_{x,\uparrow}(t) c^{\dagger}_{0,\uparrow}(0)\rangle^{(T)} &= e^{it(h-B)} \frac{1}{2\pi} \int\limits_{-\pi}^{\pi} d\eta\, F(\gamma;\eta) \\
&\left[\det\left(\hat{\mathcal{I}} + \gamma\, \hat{\mathcal{Q}}(\eta) - \gamma\, \hat{\mathcal{R}}^{(+)}(\eta)\right) + (G-1)\det\left(\hat{\mathcal{I}} + \gamma\, \hat{\mathcal{Q}}(\eta)\right)\right],
\end{aligned} \tag{43}$$

where $\hat{\mathcal{I}}$ is the unit operator on the interval $[-\pi, \pi]$, and

$$\hat{\mathcal{Q}}(\eta) \equiv \hat{\mathcal{V}}(\eta) - \frac{1-\cos\eta}{2}\, G\, \hat{\mathcal{R}}^{(-)}. \tag{44}$$

The integral operators $\hat{\mathcal{V}}(\eta)$, $\hat{\mathcal{R}}^{(-)}$, $\hat{\mathcal{R}}^{(+)}(\eta)$ (the latter two are of rank one) possess kernels

$$\begin{aligned}
\mathcal{V}(\eta|k,k') &= \frac{\ell_+(\eta|k)\,\ell_-(k') - \ell_-(k)\,\ell_+(k')}{2\pi \tan(\frac{k-k'}{2})}, \\
\mathcal{R}^{(-)}(k,k') &= \frac{1}{2\pi}\, \ell_-(k)\, \ell_-(k'), \\
\mathcal{R}^{(+)}(\eta|k,k') &= \frac{1}{\pi(1-\cos\eta)}\, \ell_+(\eta|k)\, \ell_+(\eta|k').
\end{aligned} \tag{45}$$

The representations for the correlators $\langle c^{\dagger}_{x,\downarrow}(t) c_{0,\downarrow}(0)\rangle^{(T)}$ and $\langle c_{x,\downarrow}(t) c^{\dagger}_{0,\downarrow}(0)\rangle^{(T)}$ can be obtained from (43) by inversing the sign of the external field, $B \to -B$.

In the equal-time case, $t = 0$, the integrals over η in (43) can be taken explicitly. For $t = 0$ and $x \neq 0$ one has

$$\begin{aligned}
\langle c^{\dagger}_{x,\uparrow} c_{0,\uparrow}\rangle^{(T)} &= -\langle c_{-x,\uparrow} c^{\dagger}_{0,\uparrow}\rangle^{(T)} = \\
&= \det\left(\hat{\mathcal{I}} + (\gamma-1)\hat{v} + \hat{r}\right) - \det\left(\hat{\mathcal{I}} + (\gamma-1)\hat{v}\right),
\end{aligned} \tag{46}$$

where the kernels of integral operators $\hat{v}$ and $\hat{r}$ are

$$v(k,k') = -\sqrt{\vartheta(k)}\,\frac{\sin(|x|\frac{k-k'}{2})}{2\pi\,\tan(\frac{k-k'}{2})}\,\sqrt{\vartheta(k')},$$
$$r(k,k') = \sqrt{\vartheta(k)}e^{-ix\frac{k+k'}{2}}\sqrt{\vartheta(k')}. \tag{47}$$

For $t=0$ and $x=0$ one has

$$\langle c^{\dagger}_{x,\uparrow}c_{x,\uparrow}\rangle^{(T)} = \frac{1}{2\pi}\int\limits_{-\pi}^{\pi} dk\,\vartheta(k),$$
$$\langle c_{x,\uparrow}c^{\dagger}_{x,\uparrow}\rangle^{(T)} = 1-\frac{\gamma}{2\pi}\int\limits_{-\pi}^{\pi} dk\,\vartheta(k), \tag{48}$$

and therefore, e.g.,

$$\langle c_{x,\uparrow}c^{\dagger}_{x,\uparrow}\rangle^{(T)} = 1-\left(\langle c^{\dagger}_{x,\uparrow}c_{x,\uparrow}\rangle^{(T)}+\langle c^{\dagger}_{x,\downarrow}c_{x,\downarrow}\rangle^{(T)}\right), \tag{49}$$

in agreement with the relation (30).

Consider now the temperature correlation functions of the density operators. For these correlation functions the following representation is valid

$$\langle n_{x,\alpha}(t)n_{0,\beta}(0)\rangle^{(T)} = \frac{1}{2\pi}\int\limits_{-\pi}^{\pi} d\eta\,\Phi^{(\alpha,\beta)}(\gamma;\eta)$$
$$\frac{2}{1-\cos\eta}\left[\det\left(\hat{\mathcal{I}}+\gamma\,\hat{\mathcal{U}}(\eta)\right)-\det\left(\hat{\mathcal{I}}+\gamma\,\hat{\mathcal{Q}}(\eta)\right)\right], \tag{50}$$

where

$$\Phi^{(\uparrow,\uparrow)}(\gamma;\eta) = \frac{\gamma-1}{\gamma^2}+2\pi\Delta(\eta)\,\frac{1}{\gamma^2},$$
$$\Phi^{(\uparrow,\downarrow)}(\gamma;\eta) = \Phi^{(\downarrow,\uparrow)}(\gamma;\eta) = -\frac{\gamma-1}{\gamma^2}+2\pi\Delta(\eta)\,\frac{\gamma-1}{\gamma^2},$$
$$\Phi^{(\downarrow,\downarrow)}(\gamma;\eta) = \frac{\gamma-1}{\gamma^2}+2\pi\Delta(\eta)\,\left(\frac{\gamma-1}{\gamma}\right)^2, \tag{51}$$

and $\Delta(\eta)$ is the 2π-periodic delta-function. The integral operator $\hat{\mathcal{U}}(\eta)$ possesses the kernel

$$\mathcal{U}(\eta|k,k') = \frac{\ell_+(\eta|k)\,\ell_-(k')-\ell_-(k)\,\ell_+(k')}{2\pi\sin(\frac{k-k'}{2})}, \tag{52}$$

and the integral operator $\hat{\mathcal{Q}}(\eta)$ is defined in (44). Let us note that the contributions to the correlators (50) containing the delta-function $\Delta(\eta)$ in the quantities

$\Phi^{(\alpha,\beta)}(\gamma;\eta)$ admit more explicit form. Indeed, only the Fredholm minors up to the second order contribute to the correlators in these terms. Let us denote this contribution (up to a numerical factor depending on spins) as $\mathcal{G}(x,t;h,B)$. One has

$$\begin{aligned}\mathcal{G}(x,t;h,B) &:= \frac{2}{1-\cos\eta}\left[\det\left(\hat{\mathcal{I}}+\gamma\,\hat{\mathcal{U}}(\eta)\right)-\det\left(\hat{\mathcal{I}}+\gamma\,\hat{\mathcal{Q}}(\eta)\right)\right]\Big|_{\eta=0}\\ &= \left(\frac{\gamma}{2\pi}\int\limits_{-\pi}^{\pi}dk\,\vartheta(k)\right)^2-\left|\frac{\gamma}{2\pi}\int\limits_{-\pi}^{\pi}dk\,\vartheta(k)\,e^{it\varepsilon(k)-ixk}\right|^2\\ &+\left(\frac{1}{2\pi}\int\limits_{-\pi}^{\pi}dk\,e^{-it\varepsilon(k)+ixk}\right)\left(\frac{\gamma}{2\pi}\int\limits_{-\pi}^{\pi}dk\,\vartheta(k)\,e^{it\varepsilon(k)-ixk}\right).\end{aligned}\tag{53}$$

Other contributions to the correlators (terms independent on η in $\Phi^{(\alpha,\beta)}(\gamma;\eta)$) could not be simplified in the general case since the integral over η cannot be taken in the closed form. One can, however, consider some combinations of the correlators (50) in which these terms cancel; e.g.,

$$\langle(n_{x,\uparrow}(t)+n_{x,\downarrow}(t))\,(n_{0,\uparrow}(0)+n_{0,\downarrow}(0))\rangle^{(T)}=\mathcal{G}(x,t;h,B).\tag{54}$$

In the equal-time case the integral over η in (50) can be taken for all the contributions. Remarkably that in the case $t=0$ and $x\neq 0$ the correlation functions are determined by the function $\mathcal{G}(x,t;h,B)$ only. In this case one has

$$\langle n_{x,\alpha}n_{0,\beta}\rangle^{(T)}=\phi^{(\alpha,\beta)}\left\{\left(\frac{\gamma}{2\pi}\int\limits_{-\pi}^{\pi}dk\,\vartheta(k)\right)^2-\left|\frac{\gamma}{2\pi}\int\limits_{-\pi}^{\pi}dk\,\vartheta(k)\,e^{-ixk}\right|^2\right\}\tag{55}$$

where

$$\phi^{(\uparrow\uparrow)}=\frac{1}{\gamma^2},\qquad \phi^{(\uparrow\downarrow)}=\phi^{(\downarrow\uparrow)}=\frac{\gamma-1}{\gamma^2},\qquad \phi^{(\downarrow\downarrow)}=\left(\frac{\gamma-1}{\gamma}\right)^2.\tag{56}$$

In the case $t=0$ and $x=0$ not only the function $\mathcal{G}(x,t;h,B)$ contributes to the correlators. The result is

$$\begin{aligned}\langle n_{x,\uparrow}n_{x,\uparrow}\rangle^{(T)}&=\langle n_{x,\uparrow}\rangle^{(T)}=\frac{1}{2\pi}\int\limits_{-\pi}^{\pi}dk\,\vartheta(k),\\ \langle n_{x,\downarrow}n_{x,\downarrow}\rangle^{(T)}&=\langle n_{x,\downarrow}\rangle^{(T)}=\frac{\gamma-1}{2\pi}\int\limits_{-\pi}^{\pi}dk\,\vartheta(k),\\ \langle n_{x,\uparrow}n_{x,\downarrow}\rangle^{(T)}&=\langle n_{x,\downarrow}n_{x,\uparrow}\rangle^{(T)}=0.\end{aligned}\tag{57}$$

Let us note that the obtained results for the correlation functions (43) and (50) has the proper "one-component limit". Indeed, in the limit $B\to-\infty$,

$h \to -\infty$ ($h - B = h_0$ is fixed) one has a free fermion model (of fermions with spin $\uparrow$). Since $\gamma = 1$ in this limit, the representations (43) and (50) become in fact trivial reproducing the well-known results for the correlators of free fermions on the lattice. Also for the only non-vanishing correlator of density operators, $\langle n_{x,\uparrow}(t) n_{0,\uparrow}(0) \rangle^{(T)}$, one gets in this limit the expression for the correlator of third local spin components in the $XX0$ spin chain [23, 21].

In conclusion, we would like to stress that the integral operators involved into the representations (43) and (50) are of the form usual for integrable models, i.e., they are "integrable integral operators" [6, 13]. This fact is important for constructing the corresponding matrix Riemann-Hilbert problem and for deriving integrable partial differential equations for the correlators. This, in turn, will make possible the evaluation of different (e.g., large time and distance) asymptotics of the correlators considered.

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