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THE SPECIFIC PROPERTIES OF UNSTABLE NEUTRINOS MIXING

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ABSTRACT

New specific properties of the theory of unstable neutrinos mixing were derived, which are different from the properties of the usual theory of neutrinos mixing, which was based on the assumption that all neutrinos are absolutely stable.

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0. The problem of neutrinos mixing is very essential for some intriguing problems of modern astrophysics, cosmology and elementary particle physics and was intensively investigated from theortical and experimental points of view (see, for example [1] and references therein). The neutrinos mixing is possible only if neutrinos have the nonzero masses, and this assumption is true in some modern theoretical models. The usual theory of neutrinos mixing based on the assumption that neutrinos with nonzero masses are *absolutely stable*. But this assumption is not necessary from the theoretical point of view, contrary to this the usual models predict instability (in any case for ν_μ, ν_τ neutrinos), The assumption of the stability of the neutrinos also didnot follows from the experimental data (one of the best estimates for the mean life times of neutrinos follow from the experimental data of the supernova SN 1987A neutrinos [2]). I shall prove that the theory of unstable neutrinos mixing is *different* from the usual theory, which was based on the assumption of the absolute stability of the neutrinos. Moreover, the theory of unstable neutrinos mixing predicts some very specific properties of neutrinos mixing, which could be tested by experiments. In this short report I shall give only the standing points of the problem, the main statements of the theoretical results, but not give corresponding mathematical proofs, which shall be published separately.

1. For simplicity we will limit ourselves here to the case of neutrinos with two flovers, but the general theory can be constructed for many flover case as well. So, let ν_α, ν_β are neutrinos with flovers α and β (for example, $\nu_\alpha = \nu_e$, $\nu_\beta = \nu_\mu$), and $\bar{\nu}_\alpha$. $\bar{\nu}_\beta$ are corresponding antineutrinos. Usual [1] theory of neutrinos mixing assumed that all neutrinos are absolutely stable and have nonzero masses m_{ν_α}, m_{ν_β}, $m_{\bar{\nu}_\alpha}$, $m_{\bar{\nu}_\beta}$. From the absolute stability of neutrinos (antineutrinos) follows directly, that if conservation of flovers or lepton quantum numbers is violated, then the definite masses have not ν_α, ν_β, $\bar{\nu}_\alpha$. $\bar{\nu}_\beta$, but corresponding *superpositions* ν_1, ν_2, which connect with ν_α, ν_β (or with $\bar{\nu}_\alpha$. $\bar{\nu}_\beta$) by *unitary* transformations

$$\begin{cases} |\nu_\alpha >= cos\theta|\nu_1 > + sin\theta|\nu_2 > \\ |\nu_\beta >= -sin\theta|\nu_1 > + cos\theta|\nu_2 > \end{cases} \tag{1}$$

and corresponding expressions in the case of $\bar{\nu}_\alpha$. $\bar{\nu}_\beta$. From (1) follows such time evolution

$$\begin{cases} |\nu_\alpha(t) >= e^{-iHt}|\nu_\alpha >= cos\theta e^{-iE_1t}|\nu_1 > \\ + sin\theta e^{-iE_2t}|\nu_2 >, \\ |\nu_\beta(t) >= e^{-iHt}|\nu_\beta >= -sin\theta e^{-iE_1t}|\nu_1 > \\ + cos\theta e^{-iE_2t}|\nu_2 > \end{cases} \tag{2}$$

where $E_{1,2} = \sqrt{m_{1,2}^2 + k^2}$, and k is a momentum. From (2) follows all of the usual theory of neutrinos mixing. One specific prediction of the usual theory is that

$$\begin{cases} p_{\alpha,\beta}(t) =< \nu_\alpha|e^{-iHt}|\nu_\beta >= p_{\beta,\alpha}(t) =< \nu_\beta|e^{-iHt}|\nu_\alpha > \\ P_{\alpha,\beta}(t) = |p_{\alpha,\beta}(t)|^2 = P_{\beta,\alpha}(t) = |p_{\beta,\alpha}(t)|2 \end{cases} \tag{3}$$

In the usual theory of the stable neutrinos mixing the parameters m_1, m_2, θ are *independent*.

But if the neutrinos (antineutrinos) are *unstable*, then the theory of neutrinos mixing changes essentially. First of all, if ν_1, ν_2 are *unstable* states and have not different *additional* discrete quantum numbers, then $|\nu_1 >$ and $|\nu_2 >$ must be *nonorthogonal*

$$< \nu_1|\nu_2 > \neq 0 \tag{4}$$

From this follows directly that $\nu_\alpha,\ \nu_\beta,\ \bar{\nu}_\alpha.\ \bar{\nu}_\beta$ and $\nu_1,\ \nu_2$ *cannot* be connected with *unitary* transformation of type (1). From this fundamental fact was derived (see later) that the parameters of general superpositions (*not unitary*),(which connect $\nu_\alpha,\ \nu_\beta,\ \bar{\nu}_\alpha.\ \bar{\nu}_\beta$ with $\nu_1,\ \nu_2$) and the masses $m_1,\ m_2$ and the widthes $\Gamma_1,\ \Gamma_2$ of corresponding mass distributions are *nonindependent*, contrary to the independence of the parameters $(m_1,\ m_2,\ \theta)$ of the usual theory. It was also derived that in the new rigorous theory of *unstable* neutrinos mixing some main statements of the usual theory of the *stable* neutrinos mixing were violated.

2. If ν_1 and ν_2 are unstable particles and have not new additional discrete quantum numbers, then the general theory of the time evolution of the unstable neutrinos with definite flavor quantum numbers $(\nu_e,\ \nu_\mu,\ \nu_\tau)$ or with different lepton quantum numbers $(\nu_\alpha,\ \bar{\nu}_\alpha;\ \nu_\alpha \neq \bar{\nu}_\alpha)$ is analogical to the theory of time evolution of $K^0 - \bar{K}^0$ $(B^0 - \bar{B}^0,\ D^0 - \bar{D}^0)$ with CP-violation taken into account. From this point of view the unstable neutrinos ν_1 and ν_2 are analogical to $K_S,\ K_L$, the neutrinos with different flavors $(\nu_e,\ \nu_\mu,\ \nu_\tau)$ are analogical to $K_1^0,\ K_2^0$, and $\nu_\alpha,\ \bar{\nu}_\alpha$ are analogical to $K^0,\ \bar{K}^0$. The violation of the flavor conservation law is analogical to the CP-violation and the violation of the leptonic quantum numbers is analogical to the strangness and CP-violation together. The usual theory of time evolution of kaon system with CP-violation is the phenomenological Lee–Oehme–Yang (LOY) theory [3], which was based on the famous Weisskopf-Wigner (W-W) approximation. In my previous works (see, for example, [4] and references therein) on the CP-violation problem I have derived in the rigorous theory some new and unexpected results beyond the approximate LOY theory. These are general results for the time evolution of a multilevel unstable physical system in the presence of the violation of some discrete symmetry (CP-violation, flavor conservation law violation, leptonic conservation law violation,...). I shall use in this report this general theory for the theory of the unstable neutrinos mixing.

3. Let $|\nu_\alpha>$ and $|\nu_\beta>$ are the eigenvectors of the flavor quantum number with different flavor eigenvalues

$$\begin{cases} F|\nu_\alpha>=f_\alpha|\nu_\alpha>, F|\nu_\beta>=f_\beta|\nu_\beta> \\ f_\alpha \neq f_\beta, <\nu_\alpha|\nu_\beta>=0 \end{cases} \tag{5}$$

or let $|\nu_\alpha>$ and $|\bar{\nu}_\alpha>$ are the eigenvectors of the lepton quantum number with eigenvalues $+1$ and -1 (we assume that $|\nu_\alpha> \neq |\bar{\nu}_\alpha>$)

$$\begin{cases} L|\nu_\alpha>=+1|\nu_\alpha>, L|\bar{\nu}_\alpha>=-1|\bar{\nu}_\alpha> \\ <\nu_\alpha|\bar{\nu}_\alpha>=0 \end{cases} \tag{6}$$

Let H be the full Hamiltonian including all involved interactions, which violate flavor and/or lepton number conservation laws: $[H,F]\neq 0,\ [H,L]\neq 0$. In this case $|\nu_\alpha>.\ |\nu_\beta>, |\bar{\nu}_\alpha>$ and $|\bar{\nu}_\beta>$ cannot be the eigenvectors of H and thus they must be *unstable*.

Let us start with the case of $|\nu_\alpha>, |\bar{\nu}_\alpha>$ neutrinos. This case, where the leptonic conservation law is violated, is quite analogical to the $K^0 - \bar{K}^0$ case with CP-violation. Let $|\phi_n(E)>$ be the complete orthogonal set of eigenvectors of the full Hamiltonian

$$\begin{cases} H|\phi_n(E)>=E|\phi_n(E)>, \\ <\phi_n(E)|\phi_{n'}(E')>=\delta_n^{n'}\delta(E-E') \end{cases} \tag{7}$$

of the absolute continuous spectrum. Let we have such decompositions

$$\begin{cases} |\bar{\nu}_\alpha>=\int_{SpecH}\sum_n c_{\alpha,n}(E)|\phi_n(E)>dE \\ |\bar{\nu}_\alpha>=\int_{SpecH}\sum_n c_{\bar{\alpha},n}(E)|\phi_n(E)>dE \end{cases} \tag{8}$$

and corresponding decompositions

$$\left\{ \begin{array}{l} |\nu_1 >= \int_{SpecH} \sum_n c_{1,n}(E)|\phi_n(E)> dE \\ |\nu_2 >= \int_{SpecH} \sum_n c_{2,n}(E)|\phi_n(E)> dE \end{array} \right. \tag{9}$$

By considerations , which are analogical to [4], we obtain such results. From the assumed CPT-invariance follows

$$\left\{ \begin{array}{l} \omega_\alpha(E) = \sum_n |c_{\alpha,n}(E)|^2 = \omega_{\bar\alpha}(E) = \sum_n |c_{\bar\alpha,n}(E)|^2 \\ p_{\alpha\alpha}(t) =< \alpha|\alpha(t) >= p_{\bar\alpha\bar\alpha}(t) =< \bar\alpha|\bar\alpha(t)> \end{array} \right. \tag{10}$$

By taking into account (10) we obtain analogical to [4] such

Main Theorem (M.Th.). *If the leptonic conservation law is violated ($[H,L] \neq 0$), then there donot exist any superpositions $|\nu_1>$, $|\nu_2>$ of $|\nu_\alpha>$, $|\bar\nu_\alpha>$*

$$\left\{ \begin{array}{l} |\nu_1 >= p|\nu_\alpha> + q|\bar\nu_\alpha>, |\nu_2 >= p|\nu_\alpha> - q|\bar\nu_\alpha>; |p|^2 + |q|^2 = 1 \\ <\nu_1|\nu_2> = <\nu_2|\nu_1> = |p|^2 - |q|^2 (CPT) \\ <\nu_1|\nu_2> = <\nu_2|\nu_1> = |p|^2 - |q|^2 \neq 0 ([H,L] \neq 0) \end{array} \right. \tag{11}$$

such that

A. they donot regenerate to each other in the vacuum,

B. they have a single pole (Breit–Wigner) energy distributions and the simple exponential decay laws.

The main statements of the M.Th. *directly* conradict to the corresponding main statements of the approximate LOY theory. Now I formulate the corresponding comments to the M.Th.

*Comment*1. The part A. of the M.Th. is true for *any* model interaction, which induced the violation of the leptonic quantum number.

*Comment*2. The M.Th. is true for *any* mass distributions of ν_1, ν_2 (ν_α, $\nu_{\bar\alpha}$).

*Comment*3. The essential difference between the predictions of the usual LOY theory and the rigorous theory beyond W-W approximation is that the LOY predicts that

$$p_{\alpha\bar\alpha}(t)/p_{\bar\alpha\alpha}(t) = \frac{p^2}{q^2} = const \neq 1 \tag{12}$$

where $p_{\alpha\bar\alpha}(t) =< \nu_\alpha|\nu_{\bar\alpha}(t)>$, $p_{\bar\alpha\alpha}(t) =< \nu_{\bar\alpha}|\nu_\alpha(t)>$ are the vacuum regeneration amplitudes, while the rigorous theory predicts that the ratio in (12) must be time *depenedent*

$$p_{\alpha\bar\alpha}(t)/p_{\bar\alpha\alpha}(t) \neq const \tag{12a}$$

The statement (12a) also follows from the very general theorem, which I shall formulate later. The statements (12),(12a) evidently contradict to the prediction of the type (3) of the usual theory of the *stable* neutrinos mixing.

Now for the case of violation flavor quantum numbers it is true such analog of the M.Th. without part B.

Main Theorem 2. *If the flavor conservation law is violated $[H,F]\neq 0$, then there donot exist any superpositions of $|\nu_1>$, $|\nu_2>$ of $|\nu_\alpha>$ and $|\nu_\beta>$*

$$\left\{\begin{array}{l} |\nu_1>=a|\nu_\alpha>+b|\nu_\beta>; |\nu_2>=c|\nu_\alpha>+d|\nu_\beta> \\ |a|^2+|b|^2=1=|c|^2+|d|^2 \\ <\nu_1|\nu_2>=a^*c+b^*d\neq 0,([H,F]\neq 0) \end{array}\right. \tag{13}$$

such that they donot regenerate to each other in the vacuum.

The corresponding $Comments$ to the M.Th.2 are analogical to the $Comments$ on the M.Th., in particular

$$p_{\alpha\beta}(t)/p_{\beta\alpha}(t)\neq const \tag{12b}$$

The essential statements (12a),(12b) follow also from such very General Theorem, which was proved early [4] in the investigations of the CP-violation problem:

General Theorem. *Let $|\alpha>$ and $|\beta>$ are two unstable particles (states). If the ratio*

$$p_{\alpha\beta}(t)/p_{\beta\alpha}(t)=const \tag{14}$$

is time independent, then

$$|p_{\alpha\beta}(t)|^2=|p_{\beta\alpha}(t)|^2 \tag{15}$$

This General Theorem is essential also for testing the T-invariance.

3. New predictions (12a),(12b) give unexpected and unique possibility to test the tails of the energy (mass) distributions of the unstable neutrinos ν_1, ν_2. In the natural model assumption

$$\left\{\begin{array}{l} c_{1,n}(E)=\sqrt{\frac{\Gamma_1}{\pi}}\frac{A_{1,n}(\nu_1\rightarrow(n))\xi_{1,n}(E)}{\xi_{1,n}(E_1-i\Gamma_1)(E-E_1+i\Gamma_1)} \\ c_{2,n}(E)=\sqrt{\frac{\Gamma_2}{\pi}}\frac{A_{2,n}(\nu_2\rightarrow(n))\xi_{2,n}(E)}{\xi_{2,n}(E_2-i\Gamma_2)(E-E_2+i\Gamma_2)} \end{array}\right. \tag{16}$$

where $\xi_{1,n}(E)$ and $\xi_{2,n}(E)$ are the analytical "preparation" functions and $A_{1,n}$ and $A_{2,n}$ are the partial amplitudes for the decay of ν_1 and ν_2 in the common decay channel (n). Within this model assumption (16) possible to obtain, by neglecting very small nonexponential terms [5] of the order $\Gamma_{1,2}/E_{1,2}<<1$, such expressions for the vacuum regeneration amplitudes

$$\left\{\begin{array}{l} p_{\alpha\bar{\alpha}}(t)\backsimeq\frac{1}{4p^*q}\Big[[exp(-iE_1t-\Gamma_1|t|)[1+\frac{2i\sqrt{\Gamma_1\Gamma_2}}{(E_2-E_1)+i(\Gamma_1+\Gamma_2)} \\ \sum_n A_{1,n}A^*_{2,n}\frac{\xi^*_{2,n}(E_1-i\Gamma_1)}{\xi^*_{2,n}(E_2-i\Gamma_2)}]-exp(-iE_2t-\Gamma_2|t|) \\ [1-\frac{2i\sqrt{\Gamma_1\Gamma_2}}{(E_2-E_1)-i(\Gamma_1+\Gamma_2)}\sum_n A^*_{1,n}A_{2,n}\frac{\xi^*_{1,n}(E_2-i\Gamma_2)}{\xi^*_{1,n}(E_1-i\Gamma_1)}]]] \end{array}\right. \tag{17a}$$

and

$$\left\{\begin{array}{l} p_{\bar{\alpha}\alpha}(t)\backsimeq\frac{1}{4pq^*}\Big[[exp(-iE_1t-\Gamma_1|t|)[1-\frac{2i\sqrt{\Gamma_1\Gamma_2}}{(E_2-E_1)+i(\Gamma_1+\Gamma_2)} \\ \sum_n A_{1,n}A^*_{2,n}\frac{\xi^*_{2,n}(E_1-i\Gamma_1)}{\xi^*_{2,n}(E_2-i\Gamma_2)}]-exp(-iE_2t-\Gamma_2|t|) \\ [1+\frac{2i\sqrt{\Gamma_1\Gamma_2}}{(E_2-E_1)+i(\Gamma_1+\Gamma_2)}\sum_n A^*_{1,n}A_{2,n}\frac{\xi^*_{1,n}(E_2-i\Gamma_2)}{\xi^*_{1,n}(E_1-i\Gamma_1)}]]] \end{array}\right. \tag{17b}$$

From (17a,b) we can see that endeed it is true the statement (12a). Moreover, as follows from (17a,b), one has in the regeneration amplitues interesting information not only on the energy difference $(E_2 - E_1)$, as in the approximate LOY theory, but also on the tails (!) of the energy distributions (from the terms $\xi^*_{1,n}(E_2 - i\Gamma_2)$ and $\xi^*_{2,n}(E_1 - i\Gamma_1)$) and on the decay amplitudes $A_{1,n}$, $A_{2,n}$. The analogical to (17a,b) expressions follow for the vacuum regeneration amplitudes $p_{\alpha\beta}(t)$ and $p_{\beta\alpha}(t)$. From (17a,b) follows that the new terms of the rigorous theory for the vacuum regeneration amplitudes are proportional to the overlapping of the energy distributions of the unstable $|\nu_1>$ and $|\nu_2>$. The explanation of the Sun neutrinos deficit by the neutrinos mixing [1] assumed such overlapping.

4. Contrary to the usual theory of the stable neutrinos mixing, for which the energies $E_{1,2}$ and the mixing angles θ are *independent* parameters, in the rigorous theory of the *unstable* neutrinos mixing all parameters ($E_{1,2}$, $\Gamma_{1,2}$, p, q, $A_{1,n;2,n}$ and the preparation functions $\xi_{1,2}(E)$– for the case of ν_α, $\nu_{\bar{\alpha}}$ mixing) are *not independent* and connects through the unitarity relations of the Bell-Steinberger type

$$\begin{cases} (|p|^2 - |q|^2)[(E_1 - E_2) + i(\Gamma_1 + \Gamma_2)] \\ = 2i\sqrt{\Gamma_1\Gamma_2}\sum_n A^*_{1,n}A_{2,n}\frac{\xi_{2,n}(E_1 - i\Gamma_1)}{\xi_{2,n}(E_2 - i\Gamma_2)} \end{cases} \tag{18}$$

For the prelimanary version of the results of this work see [6].

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