PDMI PREPRINT — 6/1997

QUANTUM THEORY OF UNSTABLE PARTICLES AND RELATIVITY

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March 12, 1997

ABSTRACT

A new phenomena on connection between Quantum Theory of unstable elementary particles and Relativity was derived. New exact expression for the mean life time of the unstable elementary particles with nonzero momentum was derived, which is different from the usually used. Some proposals for the critical experiments were discussed

0. As we know now almost all elementary particles (and even the fundamental particles-leptons and quarks) are *unstable* for spontanious decay. Only electron, positron and γ quants (photons) are stable as assumed by the modern theoretical point of view. From the experimental point of view even for the electron (positron) we know only such estimate for its mean life time $\tau_e > 2.7 * 10^{+27} yr.$ [1]. At the same time usually unstable elementary particles were investigated theoretically as *quasistable* and such approach based on the fact that the usual elementary particles, which decay is induced by electromagnetic and weak interactions, have the "infinite" mean life times in the scale of strong interactions, which induced the masses of these elementary particles. In another words it means that for the usual elementary particles $2\Gamma/m_0 << 1$, where m_0 is the "mass" of the unstable particle and Γ is the width of the mass distribution of the unstable particle, which corresponds to the mean life time τ according to $\tau = (2\Gamma)^{-1}$. For example, $2\Gamma_{\pi^+}/m_{\pi^+} \backsimeq 10^{-15}$, but $2\Gamma_{\pi^0}/m_{\pi^0} \backsimeq 10^{-7}$, and for particles-resonances situation is very different. For example, for neutral gauge boson Z we have [1] $2\Gamma_Z/m_Z \backsimeq 3 * 10^{-2}$ and for ρ meson even bigger $2\Gamma_\rho/m_\rho \backsimeq 2 * 10^{-1}$.

As was pointed above, contrary to the fact of the unstability of the elementary particles usually theoretical investigations neglect by this and investigate in main they in the limit $2\Gamma/m_0 \to 0$. For example, the scattering of unstable elementary particles (like π meson) was investigated as scattering of *stable* particles. This was founded on the base that $2\Gamma/m_0 << 1$, and on the base that the usual stationary quantum theory of scattering (S matrix approach) assumed that $|in>$ and $|out>$ states are states of *stable* particles (but see the nonstationary quantum theory of the wave packets scattering in [2] and references therein). Of course, it is impossible to neglect by the unstability of the elementary particles if we investigate the decay of the unstable particles. But usually in the investigations of the decay was used the famous Weisskopf- Wigner (WW) approximation, which founded the exponential decay law of unstable particles and corresponds again to the limit $2\Gamma/m_0 \to 0$. It is natural interest to the rigorous theory, in which we investigate theory beyond the limit $2\Gamma/m_0 \to 0$. This rigorous theory was derived new phenomena like the *nonexponentiality* (see [3] and references therein) of the decay law, Quantum Zeno Effect (QZE)- "Man made Physics" (see [4] and references therein). In this paper I investigate within rigorous quantum theory a new phenomena of the connection between the Quantum Theory and the Relativity, which is true even for the *exponential* decay law.

1. I shall start with the simplest example. Usually famous Pauli's prediction [5] of neutrino founded in such way. In 1914 Chadwick [6] was derived experimentally that the electrons in the β radioactive decay have the *continious* spectrum of energy. It was looked as direct contradiction to the fundamental energy-momentum conservation law within special relativity theory, if the β radioactive decay is two-body decay. Indeed, if the β radioactive atomic nucleus (Z) is in the rest (with momentum $k = 0$) and has the *fixed* (*deterministic*) mass (energy), as have also the daughter stable nucleus ($Z - 1$) and the electron, then the relativistic energy-momentum conservation law states that

$$\begin{cases} \vec{k}_{(Z-1)} + \vec{k}_e = 0 \Longrightarrow |\vec{k}_{(Z-1)}| = |\vec{k}_e| = k \\ m_{(Z)} = \sqrt{m^2_{(Z-1)} + k^2} + \sqrt{m_e^2 + k^2} \end{cases} \tag{1}$$

From (1) it evidently follows that the electron *must* has the monochromatic (δ- function) energy spectrum in the assumed two-body β radioactive decay. On the base of this contradiction Bohr in 1932 [7] (after construction of the Quantum Theory !) assumed that the energy conservation law was *violated* in the β decay, and he was refused from this assumption only in 1936. Contrary Pauli in 1930 [5] on the base of this contradiction assumed that the relativistic energy-momentum conservation law is absolutly true, but the β radioactive decay is not two-body, but is three-body decay with the new particle- neutrino. But the statement (1), on which were based Bohr's and Pauli's hypotheses, is *incorrect*, as was made clear within quantum theory of unstable states only later (Weisskopf- Wigner (1930) [8], Mandelstam- Tamm (1945) [9] and more rigirously Fock-Krylov (1947) [10]), because *unstable* radioactive nucleus (and ,in general, all unstable states) *cannot* has the *fixed* (deterministic) mass (energy) !!! On the base of the Fock- Krylov theorem the unstable β radioactive nucleus (Z) *must* has the continious mass (energy) distribution, which is the dynamical invariant. From this follows that the electron in the assumed two-body β decay *must* has also the continious energy spectrum in formal accordance with Chadwick's experimental result. So formally Bohr was right on the statement that the fixed energy is not conserved in the β decay -in the decay must be conserved the *distribution* of the energy, the continious energy spectrum (quantum dynamical invariant). But if it is so, why it was nessecary Pauli's prophecy? The fundamental *additional* experimental facts, which we know, are the mean life times of the β radioactive nucleas decay. So, if we assumed that the experimentally observed continious energy spectrum of the electron in the assumed $two-body$ decay connects with the Fock-Krylov theorem, then the mean life time of the β decay *must* be fantastically *small* !!! For example, for tritium β decay the corresponding mean life time must be of the order $10^{-17} sec$, which is in the crying contradiction with the experimental mean life time. Exactly this *contradiction* with the experimental values of the mean life times of the β radioactive decay founded the Pauli's prophecy statement on the existance of neutrino, but not his original foundation, which was theoretically incorrect (I not discuss here the additional Pauli's arguments, which were connected with spin). Only such genius as Pauli can derived his neutrino prophecy on the base of incorrect foundation. Even now incorrectness of this foundation was not well understood- see first sentence "Conservation laws have long been a foundation of physics and careful tests of their validity have led to imortant discoveries, such as neutrinos" from J.F.Bell and T.Damour "A new test of conservation laws and Lorentz invariance in relativistic gravity", Institut des Hautes Etudes Scientifiques preprint IHES/P/96/36, June 1996.

The consideration above demonstrates very delicate connection between Quantum Theory and Relativity. For more deep and delicate connection see part 2. of this paper. Of course, consideration above of the β decay of atomic nucleus has in main the historical and methodical interest. But at the same time this consideration is essential for the investigation one of the intriguing problem of modern physics- the problem of finite mass of neutrino. Indeed, the usual expression from the Fermi's theory [11] for the electron energy spectrum in the three-body (with neutrino) β decay based again on the energy-momentum conservation law for this three-body decay, which assumed that the unstable β radioactive nucleus has the *fixed* mass (energy) E_0 in contradiction with the main statement of the quantum theory of unstable states. Of course, the effect of unstability is in general very small, because the mean life times of β decay are large ($\Gamma/E_0 << 1$), but this effect is essential near the "end-point" $E = E_0 - m_{\bar{\nu}_e}$, because this effect changes the analytical properties of the energy spectrum of the electron (see new experimental works [12] and discussion of the negative $m^2_{\bar{\nu}_e}$ puzzle [1] in [13]). In the separate paper I shall discuss in details the effect of the unstability of the β radioactive atomic nucleus on the analytical properties of the energy β spectrum and on the methods of an estimation of neutrino mass $m_{\bar{\nu}_e} \geqq 0$.

2. Let us now investigate the problem of the determination of the mean life time of the unstable particles with momentum k, which is not zero. Usual consideration of this problem (see any text book in physics) is such. If the exponential decay law of the unstable paricle in rest $(k = 0)$ is for $t \geqq 0$

$$L(t) = \exp(-2\Gamma t) \tag{2}$$

then the decay law for unstable paricle with the momentum k is

$$L(t|k) = \exp(-2\Gamma \frac{m_0}{\sqrt{m_0^2 + k^2}} t) \tag{3}$$

where m_0 is the "mass" of the unstable particle. This expression follows directly from the relativistic kinematics (famous relativistic change of the time scale), if we assumed that the unstable particle has the *fixed* (determenistic) mass m_0. But this assumption is *incorrect* and evidently contradicts to the main statement of the quantum theory of unstable states (particles) !!! Nevertheless this (3) expression was excelent verified by the experiments and usually interpretates as the one of the best experimental proof of the special relativity, and this expression is the main foundation of all experiments with the elementary particles.

I shall prove now that the expression (3) is indeed very good approximation for usual elementary particles ($2\Gamma/m_0 << 1$) (but not for particles-resonances), but the *exact* expression of the exponential decay law is *different* from (3) as if it demonstartes the "violation" of the special relativity (!?).

PROOF: On the base of Fock-Krylov theorem [10] we have

$$p(t|k) = \int_{SpecH} \omega(E|k) \exp(-iEt) dE \tag{4}$$

where $p(t|k)$ is the survive amplitude (the amplitude of the probability that at the time t unstable particle will be in the initial nondecay state), and $\omega(E|k)$ is the conditional density of the energy distributiion of the unstable particle with the momentum k. From the fundamental expression of the relativity

$$E^2 = m^2 + k^2 \tag{5}$$

follows directly that

$$\omega(E|k)dE = \bar{\omega}(m|k)dm \tag{6}$$

where $\bar{\omega}(m|k)$ is the conditional density of the mass distribution of the unstable particle with momentum k. But the mass distribution of the elementary unstable particles is *invariant*, which is *independent* from the momentum $\bar{\omega}(m) = \bar{\omega}(m|k)$, and in general from the "preparation", from the conditions of the creation of the elementary particles. This is the fundamental assumption of the quantum theory of the elementary particles (see discussion of this fundamental assumption later).

Within usual Breit- Wigner (BW) simple one-pole mass distribution assumption we have

$$\bar{\omega}(m) = (\Gamma/\pi)[(m - m_0)^2 + \Gamma^2]^{-1} \tag{7}$$

where assumed such normalization condition

$$\int_{-\infty}^{+\infty} \bar{\omega}(m) dm = 1 \tag{7a}$$

in accordance with the used for the exponential decay the WW approximation. On the base of (4),(6),(7) we obtain

$$p(t|k) = \int_{SpecH} (\Gamma/\pi)[(m - m_0)^2 + \Gamma^2]^{-1} \exp(-i\sqrt{m^2 + k^2} t) dm \tag{8}$$

By neglecting the nonexponential terms we obtain from (8) for $t \geqq 0$

$$\begin{cases} p(t|k) \simeq \int_{-\infty}^{+\infty}(\Gamma/\pi)[(m-m_0)^2+\Gamma^2]^{-1}exp(-i\sqrt{m^2+k^2}t)dm \\ = exp(-i\sqrt{(m_0-i\Gamma)^2+k^2}t) \end{cases} \tag{9}$$

But we have

$$\begin{cases} \sqrt{(m_0-i\Gamma)^2+k^2} = \sqrt{m_0^2+k^2-\Gamma^2-2i\Gamma m_0} \\ = [(m_0^2+k^2-\Gamma^2)^2+4\Gamma^2 m_0^2]^{1/4}exp[-i\frac{1}{2}arctn(\frac{2\Gamma m_0}{m_0^2+k^2-\Gamma^2})] \end{cases} \tag{10}$$

On the base of (10) we obtain from (9)

$$\begin{cases} p(t|k) \simeq exp(-i[(m_0^2+k^2-\Gamma^2)^2+4\Gamma^2 m_0^2]^{1/4}cos[\frac{1}{2}arctn(\frac{2\Gamma m_0}{m_0^2+k^2-\Gamma^2})]t) \\ exp(-[(m_0^2+k^2-\Gamma^2)^2+4\Gamma^2 m_0^2]^{1/4}sin[\frac{1}{2}arctn(\frac{2\Gamma m_0}{m_0^2+k^2-\Gamma^2})]t) \end{cases} \tag{11}$$

From (11) we obtain such expression for the exponential term in the decay law of the unstable elementary particle with momentum k

$$L(t|k) = exp(-2[(m_0^2+k^2-\Gamma^2)^2+4\Gamma^2 m_0^2]^{1/4}sin[\frac{1}{2}arctn(\frac{2\Gamma m_0}{m_0^2+k^2-\Gamma^2})]t) \tag{12}$$

The new expression (12) is in general evidently *different* from the usually used expression (3). Only for $k=0$ the expressions (11) and (12) are the same as the standard

$$\begin{cases} p(t) = exp(-im_0 t-\Gamma t) \\ L(t) = exp(-2\Gamma t) \end{cases} \tag{13}$$

The biggest difference between usual expression (3) and the new exact (11),(12) is for $k=\Gamma$

$$\begin{cases} p(t|k) \simeq exp(-i[(m_0^4+4\Gamma^2 m_0^2]^{1/4}cos[\frac{1}{2}arctn(\frac{2\Gamma}{m_0})]t) \\ exp(-[(m_0^4+4\Gamma^2 m_0^2]^{1/4}sin[\frac{1}{2}arctn(\frac{2\Gamma}{m_0})]t) \end{cases} \tag{14}$$

and

$$L(t|k) = exp(-2[(m_0^4+4\Gamma^2 m_0^2]^{1/4}sin[\frac{1}{2}arctn(\frac{2\Gamma}{m_0})]t) \tag{15}$$

So for $k=\Gamma$ dissapiar the famous relativistic change of the time scale for the mean life time!!! Of course, this not in the least meens the "violation" of the special relativity, because the special relativity by itself was derived *within classical physics* and the usual relativistic change of time scale is true only for the motion of the stable particles of classical physics with the fixed mass- in classical physics donot exist unstable particles with the continious mass distribution, and the spontaniuos decay of the unstable states (particles) is the typical *quantum* phenomena.

If $\Gamma << m_0$ and $k >> \Gamma$, as it is true for the usual unstable elementary particles (like π mesons, but not for particles-resonances like ρ meson) with relativistic momentum k, then from (11) and (12) follow the usual results as the main term of the asymptotics for $\Gamma/m_0 \to 0$

$$\begin{cases} p(t|k) = exp(-i\sqrt{m_0^2+k^2}t)exp(-\Gamma\frac{m_0}{\sqrt{m_0^2+k^2}}t) \\ L(t|k) = exp(-2\Gamma\frac{m_0}{\sqrt{m_0^2+k^2}}t) \end{cases} \tag{16}$$

Derived above unexpected results prove that in principal the *exact* experimental investigation of the decay law of the unstable elementary parrticles with momentum $k \neq 0$, but not only the main asymptotical term (see (16)), *must* indicate the violation of the usually used (3) relativistic kinematics for the mean life time. This exact experimental result will be not the proof of the violation of the special relativity, but contrary such experimental result will essentially *confirm* the truth of the relativity not only for classical, but also for quantum physics, including the special properties of the unstable elementary particles. A priori possibility of the violation of the relativity for the unstable elementary particles is not excepted, because the axiomatics of the relativity is true within classical physics, in which all particles are absolutly stable. Such hard (see below) experiment will be interesting also, because the truth of the expressions (11),(12) will be the proof of the *fundamental* assumption of the physics of the elementary particles on the *invariance* of the mass distributions of such particles *independently* of the reactions of they creation. This *fundamental* assumption of the *identity* of the *all* elementary particles of the fixed type (for example, of *all* π^+ mesons) is the base of elementary particles physics, which was unconditionally supported for *stable* elementary particles (like all electrons in the universe) by famous Pauli's principle, but still is not absolutly proved for *unstable* elementary particles, because the application of the Pauli's principle for the unstable particles is not so *evident* [14]. In [14] it was derived that for the proof of the Pauli's principle for unstable particles we must know on the *identity* of the mass distributions for *all* unstable particles of the fixed type (for example, for all π^0 mesons). This need experiments with resolution power in mass (energy) of the odrer of the width of the mass distributions. Today experiments have not such resolution power for the usual unstable particles, but for particles-resonances such real experiments (for example in the reaction of creation of two ρ meson- twins) were proposed in [14].

Because the difference between the usually used expression (3) and the exact expression (12) is true for all times, then it is possible to do experiments for the times of the order of the mean life times, for which the experimental statistics can be sufficiently large. From this point of view such experiments have the preference before the experiments on the nonexponentiality (see record experiments of E.B.Norman team (LBL) [15] and article in "Nature" [16]), especially for the long times. The optimal experiment now (because we cannot now to measure the decay law for particles-resonances) is for π^0 meson. For $k = \Gamma_{\pi^0}$ we obtain for the ususal expression (3)

$$L_{\pi^0}(t|k = \Gamma_{\pi^0}) = exp(-2\Gamma_{\pi^0}[1 - 1,8 * 10^{-15}]t) \tag{17}$$

and for the exact expression (12)

$$L_{\pi^0}(t|k = \Gamma_{\pi^0}) = exp(-2\Gamma_{\pi^0}[1 + 1,0 * 10^{-14}]t) \tag{18}$$

We see that such experiment need to improve today accuracy of the measuremenet of the mean life time of π^0 meson by 14 orderds, and this looks as fantastics with today experimental possibilities. But it is possible to use the Mossbauer effect as filter, which gives possibility of the experiment on the decay with the filtering of the decay products (γ quants) with the resolution inside the natural line, or in another words *inside* the width of the energy (mass) distribution of the unstable nucleus. In a separate paper I shall give detail theoretical investigation of this intriguing proposal.

Another intriguing possibility, and even more interesting, is the experiment with K^0-$\bar{K}^0$ mesons, which have the big mass distribution overlapping. As was derived in my works (see, for example, [4],[17] and references therein) this unique situation give new effects beyond the approximate Lee- Oehme- Yang theory [18], which was based on the WW approximation, and these new effects are proportional to the dimensless parameter $\Gamma_L/(m_L - m_S) \simeq 10^{-3}$. This parameter is much bigger than the usual dimensless parameter for usual elementary particles $\Gamma/m_0 << 1$. In a separate paper I shall give detail theory of the interference experiment for K^0-$\bar{K}^0$, which will give the real possibility to test new expressions (11), (12) for the mean life time, and energy of the unstable particles with nonzero momentum.

ACKNOWLEDGMENTS

My big thanks to M. Gromov for the invitation (September 1996) to the Institut des Hautes Etudes Scientifiques (IHES), warm hospitality and extremly nice conditions for scientific work at IHES. My specially thanks to M. Gromov, A. Connes, M. Kontsevich for interesting discussions on the specific properties of the quantum theory of the unstable states (particles).

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